Physical States and Gauge Independence of the Energy-Momentum Tensor in Quantum Electrodynamics

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Abstract

Discussions are made on the relationship between physical states and gauge independence in QED. As the first candidate take the LSZ-asymptotic states in a covariant canonical formalism to investigate gauge independence of the (Belinfante's) symmetric energy-momentum tensor. It is shown that expectation values of the energy-momentum tensor in terms of those asymptotic states are gauge independent to all orders. Second, consider gauge invariant operators of electron or photon , such as the Dirac's electron or Steinmann's covariant approach, expecting a gauge invariant result without any restriction. It is, however, demonstrated that to single out gauge invariant quantities is merely synonymous to a gauge fixing, resulting again in use of the asymptotic condition when proving gauge independence. Nevertheless, it is commented that these invariant approaches is helpful to understand the mechanism of the LSZ-mapping and furthermore of quark confinement in QCD. As the final candidate, it is shown that gauge transformations are freely performed under the functional representation or the path integral expression on account of the fact that the functional space is equivalent to a collection of infinitely many inequivalent Fock spaces. The covariant LSZ formalism is shortly reviewed and the basic facts on the energy-momentum tensor are also illustrated.

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1 Introduction

Symmetries play an important role in physics. In the usual situation, invariance, once accepted, should be maintained throughout the story. However gauge symmetry in quantum field theory (QFT) has a rather different scenario: classically electric and magnetic fields are gauge invariant but quantization has to be carried out in terms of gauge potentials by *fixing a gauge* according to a standard recipe such as Dirac's [1] for instance. Start with a classical Lagrangian, follow the canonical procedure until setting up the Dirac bracket then utilize the corresponding principle to obtain quantum theory. Accordingly *each quantization in various gauges be carried out in a different Fock space*, so what do we mean by "gauge invariance" of the S-matrix or expectation values of observables after quantization? Moreover in order to get a representation in QFT, it is unavoidable to introduce asymptotic fields, satisfying *linear* hyperbolic field equations as well as being gauge invariant. Then how to bridge between asymptotic fields and the Heisenberg fields (the LSZ-mapping) satisfying *nonlinear* equation in a *fixed gauge*?

A way for proving gauge independence of, for instance, the S-matrix, has so far been to show it under the perturbation theory with the use of a gauge dependent photon propagator [2]. There takes part a notion of physical states, such as the on-shell condition for electron or the photon polarization condition. An ambitious trial is to introduce gauge invariant fields for electron and photon [3, 4, 5, 6], expecting a fully gauge invariant result without any conditions. The approach furthermore leads us toward the understanding of the issue of quark confinement [7]. Apart from these, the most widely adopted method of proving gauge independence is that of functional integration [8]: start with some gauge by inserting the delta-function into path integral and move to another gauge by means of the change of variables. However, as for the authors' knowledge, there seems almost no clarification of this fact.

In this paper, we first study gauge independence of the energy-momentum tensor in a covariant canonical formalism. Reasons for taking this issue are as follows:

- (a) There have been trials for proving gauge independence of the S-matrix [2] but seems very few for checking that of energy-momentum or the energy-momentum tensor. Needless to say that energy-momentum must be gauge independent and so should be the energy-momentum tensor coupled to gravity. Furthermore the energy-momentum tensor is a good object to check some invariance; since it is a composite operator so if higher order corrections are taken into account there sometimes emerge serious problems which cannot be seen in a formal discussion [9]: the self-stress problem of electron [10] or the trace anomaly [11] is well-known. Freedman et al. [12] studied that for tadpole contributions of the energy-momentum tensor in the scalar QED but they do not make any explicit calculation for other parts.
- (b) It is preferable to utilize the covariant formulation under the perturbation theory so that the covariant LSZ-formalism [13] must be suitable. It is then necessary to impose physical state conditions and check whether those give us a gauge independent result.

To make the situation clearer consider the canonical energy-momentum tensor:

$$T_{\mu\nu} \equiv \sum_a \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi^a)} \partial_\nu \phi^a - g_{\mu\nu}\mathcal{L}. \tag{1.1}$$

In terms of the (classical) Lagrangian

$$\mathcal{L}_c = \overline{\psi}\left(\frac{i}{2}\overleftrightarrow{\not{D}} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{1.2}$$

with

$$
\not{D} \equiv \gamma^\mu D_\mu , \qquad D_\mu \equiv \partial_\mu - ieA_\mu,
\\
\Phi^* \overleftrightarrow{D}_\mu \Psi \equiv \Phi^* D_\mu \Psi - (D_\mu \Phi)^* \Psi,
\tag{1.3}
$$

it reads

$$
T^c_{\mu\nu} \equiv \overline{\psi} \frac{i}{2} \gamma_\mu \overleftrightarrow{\partial}_\nu \psi - F_{\mu\rho} \partial_\nu A^\rho - g_{\mu\nu} \mathcal{L}_c \tag{1.4}
$$

which is apparently gauge variant, contrary to the Belinfante's symmetric energy-momentum tensor,

$$
\Theta^c_{\mu\nu} \equiv \frac{i}{4} \overline{\psi} \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \psi - F_{\mu\rho} F_\nu{}^\rho - g_{\mu\nu} \mathcal{L}_c \ , \tag{1.5}
$$

considered as the source of gravity, except for the scalar case in which an improvement must be necessary [9]. However there is no problem for energy-momentum: as is well-known from the process of construction, difference is given by the total derivative, $T^c_{\mu\nu} = \Theta^c_{\mu\nu} + \partial^\rho X_{[\rho,\mu]\nu}$ (with ρ, μ being antisymmetric), then

$$
P_\mu = \int d^3x \ T^c_{0\mu} = \int d^3x \ \Theta^c_{0\mu}. \tag{1.6}
$$

Therefore energy-momentum itself and the Belinfante tensor is gauge invariant and can be considered as observables classically.

The quantum Lagrangian is, due to Nakanishi and Lautrup [14],

$$
\mathcal{L} \equiv \mathcal{L}_c + \mathcal{L}_{GF} = \overline{\psi} \left(\frac{i}{2} \overleftrightarrow{\not{D}} - m \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A^\mu \partial_\mu B + \frac{\alpha}{2} B^2, \tag{1.7}
$$

with

$$
\mathcal{L}_{GF} \equiv -A^\mu \partial_\mu B + \frac{\alpha}{2} B^2, \tag{1.8}
$$

where B is an auxiliary field, called the Nakanishi-Lautrup field, and α is the gauge parameter. Although gauge has been fixed we are left with the *BRS-symmetry* [15, 16]:

$$
\delta_B A_\mu = \partial_\mu c, \quad \delta_B c = 0, \quad \delta_B B = 0, \quad \delta_B \overline{c} = -iB,
\\
\delta_B \psi = iec\psi, \quad \delta_B \overline{\psi} = -iec\overline{\psi},
\tag{1.9}
$$

with $c(\overline{c})$ being the Faddeev-Popov ghost (anti-ghost). This keeps the following Lagrangian intact:

$$
\mathcal{L} + \mathcal{L}_{FP} = \mathcal{L}_c + \mathcal{L}_{GF} + \mathcal{L}_{FP}, \tag{1.10}
$$

with

$$
\mathcal{L}_{FP} \equiv i \partial_\mu \overline{c} \partial^\mu c. \tag{1.11}
$$

Note that

$$
\mathcal{L}_{GF} + \mathcal{L}_{FP} = \delta_B \Big(i \frac{\alpha}{2} \overline{c} B - i \partial_\mu \overline{c} A^\mu \Big) . \tag{1.12}
$$

Thus gauge symmetry has been taken over by the BRS symmetry in quantum theory. The generator is called the BRS charge, Q_B;

$$
[A_\mu, Q_B] = i\delta_B A_\mu, \quad \{\psi, Q_B\} = i\delta_B \psi, \quad \ldots \tag{1.13}
$$

which gives a physical state condition:

$$
Q_B |phys\rangle_B = 0. \tag{1.14}
$$

(Here $(\{a,b\})[a,b]$ is the (anti-)commutator.) Since the canonical energy-momentum tensor is not observable even classically, we only concentrate on the Belinfante's one given as

$$\begin{aligned}\Theta_{\mu\nu} &= \Theta^c_{\mu\nu} - (A_\mu\partial_\nu B + A_\nu\partial_\mu B) + i\partial_\mu\bar{c}\partial_\nu c + i\partial_\nu\bar{c}\partial_\mu c - g_{\mu\nu}\left(\mathcal{L}_{GF} + \mathcal{L}_{FP}\right) \\ &= \Theta^c_{\mu\nu} + \delta_B\Big[-i\partial_\mu\bar{c}A_\nu - i\partial_\nu\bar{c}A_\mu - g_{\mu\nu}\left(i\frac{\alpha}{2}\bar{c}B - i\partial_\mu\bar{c}A^\mu\right)\Big]\,,\end{aligned} \tag{1.15}$$

which is, of course, BRS invariant;

$$\Big[Q_B, \Theta_{\mu\nu}\Big] = 0\ . \tag{1.16}$$

In view of (1.13) and (1.14),

$${}_B\langle phys'|\Theta_{\mu\nu}|phys\rangle_B = {}_B\langle phys'|\Theta^c_{\mu\nu}|phys\rangle_B\ . \tag{1.17}$$

So when sandwiched between *properly chosen physical* states obeying (1.14), the expectation value of the symmetric energy-momentum tensor would become gauge independent *provided that higher orders make no harmful effects.*

In order to check this, we work with the perturbative method in the covariant formalism to calculate expectation values of the energy-momentum tensor in terms of the loop expansion in §2 and convince ourselves of gauge independence (BRS invariance) to all orders. Main machinery is the Ward-Takahashi relation with the aid of the dimensional regularization which does not break the Poincaré as well as the gauge invariance and is handier than the Pauli-Villars regularization.

We then discuss about gauge invariant operators, expecting that the result would be gauge (BRS) invariant unconditionally. However, from the discussion of §3, we recognize that *picking up gauge invariant operators for basic fields, that is, for electrons and photons, is merely synonymous to gauge fixing* so that we again need physical state conditions to prove gauge independence. We argue the LSZ-mapping in terms of these invariant fields also in §3.

There is another physical state frequently adopted; the Gauss's law constraint in the $A_0 = 0$ gauge,

$$\Phi(\boldsymbol{x})|phys\rangle \equiv \left[\sum_{k=1}^{3}\left(\partial_k \boldsymbol{E}_k(\boldsymbol{x})\right) + J_0(\boldsymbol{x})\right]|phys\rangle = 0, \tag{1.18}$$

where $\boldsymbol{E}_k(J_0)$ is the electric fields (the charge density). According to the common sense in QFT [18] (1.18) implies $\Phi(\boldsymbol{x}) = 0$, but there have been a number of discussions in terms of this method. In §4 we clarify the reason by means of the functional representation. Also on account of that we can build up the path integral formula starting from the Coulomb gauge and prove gauge independence more rigorously than the previous work [8]. The final §5 is devoted to discussion. In Appendix A, we review the covariant LSZ formalism, and in Appendix B we study the violation of the Ward-Takahashi identity for the energy-momentum tensor and then perform an explicit renormalization for the energy-momentum tensor to illustrate that the usual procedure does indeed work well.

2 LSZ and the Energy-Momentum Tensor

In this section in order to examine gauge independence of the expectation value we apply the covariant LSZ formalism to the energy-momentum tensor and demonstrate that asymptotic states can indeed be interpreted as physical states and there is no harmful contribution from higher orders.

2.1 Physical States in the LSZ formalism

Start with a discussion on physical states in the LSZ formalism. Details must be seen in Appendix A. In view of the BRS invariant Lagrangian,

$$\mathcal{L} + \mathcal{L}_{FP} = \overline{\psi}\left(\frac{i}{2}\overleftrightarrow{\not{D}} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A^{\mu}\partial_{\mu}B + \frac{\alpha}{2}B^2 + i\partial_{\mu}\overline{c}\partial^{\mu}c, \tag{2.1}$$

the ghosts are free, $\Box c = \Box\overline{c} = 0$, all the time so that we can reduce physical state $|phys\rangle_B$ (1.14) to a simpler form: first decompose the total space such that [16]

$$\mathcal{V} \otimes |0\rangle_{FP} \tag{2.2}$$

where $|0\rangle_{FP}$ is the vacuum of the FP ghost sector and $\mathcal{V}$ is the remainder. $Q_B|phys\rangle_B = 0$, (1.14), then implies

$$Q_B|phys\rangle_B = Q_B\,|phys\rangle \otimes |0\rangle_{FP} = i\int d^3q\mathcal{B}_{\boldsymbol{q}}\,|phys\rangle \otimes c^{\dagger}_{\boldsymbol{q}}\,|0\rangle_{FP} = 0\ , \tag{2.3}$$

where the BRS charge has been given by

$$Q_B = i\int d^3q\left[c^{\dagger}_{\boldsymbol{q}}\mathcal{B}_{\boldsymbol{q}} - \mathcal{B}^{\dagger}_{\boldsymbol{q}}c_{\boldsymbol{q}}\right] \tag{2.4}$$

with $c^{\dagger}_{\boldsymbol{q}}(c_{\boldsymbol{q}})$ and $\mathcal{B}^{\dagger}_{\boldsymbol{q}}(\mathcal{B}_{\boldsymbol{q}})$ being the creation (annihilation) operator of the ghost and the B field respectively. From (2.3) physical state in this reduced space reads $\mathcal{B}_{\boldsymbol{q}}\,|phys\rangle = 0$, giving [14]

$$B^{(+)}(x)\,|phys\rangle = 0\ . \tag{2.5}$$

Thus throwing away the ghosts in (2.1) we begin with

$$\mathcal{L} = \overline{\psi}\left(\frac{i}{2}\overleftrightarrow{\not{D}} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A^{\mu}\partial_{\mu}B + \frac{\alpha}{2}B^2. \tag{2.6}$$

Since $B(x)$ is free, $\Box B(x) = 0$, it goes to the asymptotic field itself:$B(x) \longrightarrow Z_3^{-1/2}B^{as}(x)$, where "as" designates the asymptotic field (in or out), so that the physical state reads

$$B^{as(+)}(x)\,|phys;as\rangle = 0\ . \tag{2.7}$$

The commutation relations with respect to B,

$$[\,A_{\mu}(x)\,,B(y)\,] = -i\,\partial^x_{\mu}D(x-y)\ ,\quad [\,B(x)\,,B(y)\,] = 0\ , \tag{2.8}$$

trivially become

$$[\,A^{as}_{\mu}(x)\,,B^{as}(y)\,] = -i\,\partial^x_{\mu}D(x-y)\ ,\quad [\,B^{as}(x)\,,B^{as}(y)\,] = 0\ , \tag{2.9}$$

but those with electrons

$$[\,B(x)\,,\psi(y)\,] = e\psi(y)D(x-y)\ ,\quad [\,B(x)\,,\overline{\psi}(y)\,] = -e\overline{\psi}(y)D(x-y)\ , \tag{2.10}$$

would become

$$[\,B^{as}(x)\,,\psi^{as}(y)\,] = 0\ ,\quad [\,B^{as}(x)\,,\overline{\psi}^{as}(y)\,] = 0\ ; \tag{2.11}$$

since the interaction should fade away in the asymptotic region. If we admit (2.11) asymptotic states of electron would all be physical, that is, $\psi^{as}(y)$ is BRS invariant. As for those of photon, the B-state

$$|\boldsymbol{q};as\rangle \equiv \mathcal{B}_{\boldsymbol{q}}^{as\dagger}\,|0\rangle \ \ , \tag{2.12}$$

is physical owing to the second relation in (2.9), but

$$|\boldsymbol{q}\sigma;as\rangle \equiv \mathcal{A}_{\boldsymbol{q}\sigma}^{as\ \dagger}\,|0\rangle \ \ , \tag{2.13}$$

with $\mathcal{A}_{\boldsymbol{q}\sigma}^{as\ \dagger}\left(\mathcal{A}_{\boldsymbol{q}\sigma}^{as}\right)$ being the creation (annihilation) operator of photon needs an additional constraint to be physical. Introduce the photon wave functions, $\{h_{\boldsymbol{q}\sigma}{}^{\mu}(x)\}$, $\{f_{\boldsymbol{q}\sigma}{}^{\mu}(x)\}$, defined through

$$\Box h_{\boldsymbol{q}\sigma}{}^{\mu}(x) = f_{\boldsymbol{q}\sigma}{}^{\mu}(x)\ ; \qquad \Box f_{\boldsymbol{q}\sigma}{}^{\mu}(x) = 0; \tag{2.14}$$

those which are related each other:

$$h_{\boldsymbol{q}\sigma}{}^{\mu}(x) = \frac{1}{2}\left(\boldsymbol{\nabla}^2\right)^{-1}\left\{\left(x_0\,\partial_0 - \frac{3}{2}\right) f_{\boldsymbol{q}\sigma}{}^{\mu}(x) + g^{\mu 0} f_{\boldsymbol{q}\sigma}{}^{0}(x)\right\} \ . \tag{2.15}$$

Write the Fourier transformation as

$$f_{\boldsymbol{q}\sigma}{}^{\mu}(x) = \int\frac{d^3p}{\sqrt{(2\pi)^3 2p_0}}\,\xi_{\sigma}{}^{\mu}(\boldsymbol{p})\,\varphi_{\boldsymbol{q}}(\boldsymbol{p})\,\mathrm{e}^{-ipx}\ ; \qquad p_0 = |\boldsymbol{p}|\ , \tag{2.16}$$

where $\varphi_{\boldsymbol{q}}(\boldsymbol{p})$'s are some orthonormal set and $\xi_{\sigma}{}^{\mu}(\boldsymbol{p})$ is the photon polarization vector satisfying

$$\xi_{\sigma}{}^{\mu}(\boldsymbol{p})\,\xi_{\tau\mu}(\boldsymbol{p}) = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \equiv \eta_{\sigma\tau}\ , \tag{2.17}$$

where the repeated indices imply a summation. The physical state condition for (2.13) then gives

$$B^{as(+)}(x)\,|\boldsymbol{q}\sigma;as\rangle = [\,B^{as(+)}(x)\,,\mathcal{A}_{\boldsymbol{q}\sigma}^{as\ \dagger}\,]\,|0\rangle = \partial_{\mu} f_{\boldsymbol{q}\sigma}{}^{\mu}(x)\,|0\rangle \ \equiv \overline{f}_{\boldsymbol{q}\sigma}(x)\,|0\rangle\left(\longrightarrow 0\right)\ , \tag{2.18}$$

where use has been made of the first relation in (2.9). Thus

$$\overline{f}_{\boldsymbol{q}\sigma}(x) = \partial_{\mu} f_{\boldsymbol{q}\sigma}{}^{\mu}(x) = 0\ . \tag{2.19}$$

Note that the transversal condition of photon,

$$f_{\boldsymbol{q}\sigma}{}^{0}(x) = 0, \quad \text{as well as} \quad \sum_{l=1}^{3}\nabla_l f_{\boldsymbol{q}\sigma}{}^{l}(x) = 0\ , \tag{2.20}$$

belongs to (2.19). In terms of the momentum representation, (2.19) turns out to be

$$p_{\mu}\xi_{\sigma}{}^{\mu}(\boldsymbol{p}) = 0\ . \tag{2.21}$$

Note that there remain only two components out of 16 $\xi_{\sigma}{}^{\mu}(\boldsymbol{p})$'s; since there are 10 orthonormal conditions, (2.17), together with 4 physical state conditions, (2.21), leaving us two components. (Notations and further details should be consulted for Appendix A.)

The LSZ reduction formula of some operator $\mathcal{O}$ for photon reads, for example, as

$$\begin{aligned}
&\langle \boldsymbol{q}\sigma; out|\, T(A_\nu(y)\mathcal{O})\,|0\rangle \\
&\quad = \int d^4x \Big\{ f_{\boldsymbol{q}\sigma}^{\ \mu *}(x)\Box^x \langle 0|\, T(A_\mu(x)A_\nu(y)\mathcal{O})\,|0\rangle \\
&\qquad -(1-\alpha)\,[\,\overline{h}^{\,*}_{\boldsymbol{q}\sigma}(x)\Box^x + f_{\boldsymbol{q}\sigma}^{\ \mu *}(x)\,\partial^x_\mu]\,\langle 0|\, T(B(x)A_\nu(y)\mathcal{O})\,|0\rangle \Big\}\ ,
\end{aligned} \tag{2.22}$$

where $\overline{h}^{\,*}_{\boldsymbol{q}\sigma}(x) \equiv \partial_\mu h_{\boldsymbol{q}\sigma}^{\ \mu *}(x)$. Imposing the physical state condition (2.19) to (2.22) by the help of (2.15) and (2.20) we find that the B-containing term in the right hand side is dropped, giving a naïve amplitude consisting solely of A_μ's. The reduction for electrons can be obtained in a usual manner. Accordingly the task is to calculate the vacuum expectation value of the energy-momentum tensor,

$$\Theta^{\mu\nu} \equiv \Theta^{\mu\nu}_{\rm g} + \Theta^{\mu\nu}_{\rm m}, \tag{2.23}$$

where

$$\Theta^{\mu\nu}_{\rm g} \equiv -F^{\mu\rho}F^{\nu}_{\ \ \rho} - (A^\mu\,\partial^\nu B + A^\nu\,\partial^\mu B) - g^{\mu\nu}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A^\mu\partial_\mu B + \frac{\alpha}{2}B^2\right], \tag{2.24}$$

is the photon part and

$$\Theta^{\mu\nu}_{\rm m} \equiv \frac{1}{4}\overline{\psi} i(\gamma^\mu \overleftrightarrow{D}^\nu + \gamma^\nu \overleftrightarrow{D}^\mu)\psi - g^{\mu\nu}\overline{\psi}\left(\frac{i}{2}\overleftrightarrow{\not{D}} - m\right)\psi\ , \tag{2.25}$$

is the (gauge invariant) electron part. Remove external legs from the vacuum expectation value and multiply photon wave functions, $f_{\boldsymbol{q}\sigma}^{\ \lambda}(x)$'s, or the free spinors, $\overline{u}(\boldsymbol{k}'), u(\boldsymbol{k})$, to obtain the desired quantity; energy-momentum tensor in the physical state.

It is convenient to introduce the generating functional such that

$$\begin{aligned}
&\exp\left[iW[J_\mu, J, \overline{\eta}, \eta, \tau^{\rm m}_{\mu\nu}, \tau^{\rm g}_{\mu\nu}]\right] \equiv \exp\left[iW[\boldsymbol{J}, \boldsymbol{\eta}, \boldsymbol{\tau}]\right] \\
&\equiv \langle 0|\, T^* \exp\left[i\int d^4x\left\{J_\mu A^\mu + JB + \overline{\eta}\psi + \overline{\psi}\eta + \tau^{\rm m}_{\mu\nu}\Theta^{\mu\nu}_{\rm m} + \tau^{\rm g}_{\mu\nu}\Theta^{\mu\nu}_{\rm g}\right\}\right]|0\rangle
\end{aligned} \tag{2.26}$$

from which we obtain the connected Green's functions,

$$\begin{aligned}
&G^{\mu\nu;\lambda_1\cdots\lambda_m}_{\rm a(=g\,or\,m)}(0; x_1,\cdots,x_n; y_1,\cdots y_m, z_1,\cdots,z_m) \\
&\equiv \left[\frac{\delta}{\delta\tau^{\rm a}_{\mu\nu}(0)}\right]\left[\frac{\delta^n}{i\delta J_{\lambda_1}(x_1)\cdots i\delta J_{\lambda_n}(x_n)}\right]\frac{\delta^{2m}W[\boldsymbol{J},\boldsymbol{\eta},\boldsymbol{\tau}]}{\delta\overline{\eta}(y_1)\cdots\delta\overline{\eta}(y_m)\delta\eta(z_1)\cdots\delta\eta(z_m)}\ , \\
&= \langle 0|\, T^*\Theta^{\mu\nu}_{\rm a}(0)A_{\lambda_1}(x_1)\cdots A_{\lambda_n}(x_n)\psi(y_1)\cdots\psi(y_m)\overline{\psi}(z_1)\cdots\overline{\psi}(z_m)\,|0\rangle_{\rm conn} \\
&\quad \equiv \int\prod_{j=1}^n \frac{d^4q_j}{(2\pi)^4}\prod_{l=1}^m \frac{d^4k_l}{(2\pi)^4}\frac{d^4p_l}{(2\pi)^4}\ \exp\left[-i\sum_{j=1}^n q_jx_j - i\sum_{l=1}^m (k_ly_l - p_lz_l)\right] \\
&\qquad\qquad \times G^{\mu\nu\lambda_1\cdots\lambda_n}_{\rm a}(q_1,\cdots,q_n; k_1,\cdots,k_m,p_1,\cdots,p_m)\ ,
\end{aligned} \tag{2.27}$$

where T^* designates a covariant T-product. Calculations are performed by means of the loop expansion.

Our ingredients are summarized as follows:

- (i) The physical state conditions: for photon

$$q_\mu \xi_\sigma{}^\mu(\boldsymbol{q}) = 0 \ . \tag{2.28}$$

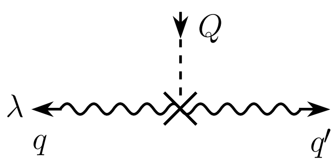
For electron

$$\begin{aligned} (\not{p} - m)\, u(\boldsymbol{p}, s) &= 0 \ , \\ \overline{u}(\boldsymbol{k}, s)\, (\not{k} - m) &= 0 \ . \end{aligned} \tag{2.29}$$

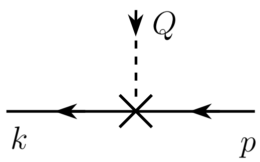
- (ii) Dimensional regularization; which preserves both gauge symmetry and the Poincaré symmetry. (Note that using a naïve cutoff breaks the situation; see Appendix B.)
- (iii) The notion of finiteness of the energy-momentum tensor [9, 12]; under which we can proceed only with the unrenormalized form. Divergences can be subtracted in a gauge invariant way in anytime. (A short discussion on renormalization is seen also in Appendix B.)

2.2 Tree Calculation

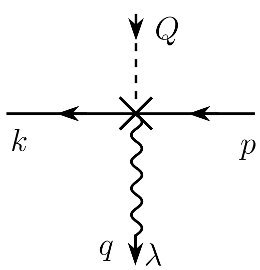
The tree graphs give us basic vertices of the Feynman rule:



$$: \quad G_{\rm g}^{\mu\nu;\lambda\kappa}(q,q')^{(0)} \equiv \frac{-i}{q^2} X^{\mu\nu\lambda\kappa}(q,q') \frac{-i}{q'^2} \ ,$$



$$: \quad G_{\rm m}^{\mu\nu}(k,p)^{(0)} \equiv \frac{i}{\not{k} - m} Y^{\mu\nu}(k+p) \frac{i}{\not{p} - m} \ , \tag{2.30}$$



$$: \quad G_{\rm m}^{\mu\nu;\lambda}(q,k,p)^{(0)}_{\rm 1PI} \equiv \frac{i}{\not{k} - m} \frac{-ie}{q^2} Z^{\mu\nu\lambda}(q) \frac{i}{\not{p} - m} \ ,$$

where the cross, "×" , and "1PI" designate the insertion of $\Theta_{\mu\nu}$ and the one-particle-irreducible part respectively. $X^{\mu\nu\lambda\kappa}(q, q')$ in (2.30) is explicitly given as

$$X^{\mu\nu\lambda\kappa}(q,q') \equiv \widetilde{X}^{\mu\nu\lambda\kappa}(q,q') - \Big(q^\lambda X^{\mu\nu\kappa}(q,q') + q'^\kappa X^{\mu\nu\lambda}(q,q')\Big) + \alpha q^\lambda q'^\kappa g^{\mu\nu} \ , \tag{2.31}$$

where

$$\widetilde{X}^{\mu\nu\lambda\kappa}(q,q') \equiv (\delta^\mu_\rho \delta^\nu_\sigma + \delta^\mu_\sigma \delta^\nu_\rho - \frac{1}{2} g^{\mu\nu} g_{\rho\sigma})(q^\rho g^{\lambda\alpha} - q^\alpha g^{\lambda\rho})(q'^\sigma \delta^\kappa_\alpha - q'_\alpha g^{\kappa\sigma}) \ , \tag{2.32}$$

comes from FF term in (2.24) thus is gauge invariant but the remaining terms are from AB and BB and then gauge variant:

$$X^{\mu\nu\kappa}(q,q') \equiv (\delta^\mu_\rho \delta^\nu_\sigma + \delta^\mu_\sigma \delta^\nu_\rho - g^{\mu\nu} g_{\rho\sigma}) q^\rho d^{\kappa\sigma}(q') \ , \tag{2.33}$$

where

$$d^{\mu\nu}(q) \equiv g^{\mu\nu} - (1-\alpha)\frac{q^\mu q^\nu}{q^2} \ , \tag{2.34}$$

is the numerator of the photon propagator,

$$D^{\mu\nu}(q) \equiv -i\frac{d^{\mu\nu}(q)}{q^2} \ . \tag{2.35}$$

Note that the transverseness of $\widetilde{X}^{\mu\nu\lambda\kappa}(q,q')$,

$$q_\lambda \widetilde{X}^{\mu\nu\lambda\kappa}(q,q') = q'_\kappa \widetilde{X}^{\mu\nu\lambda\kappa}(q,q') \ = 0 \ , \tag{2.36}$$

and the structure of gauge dependent terms: those depend on the external photon momentum as well as the index, i.e. , on q^λ, and/or q'^κ, leaving no effect owing to the physical photon condition (2.28). $G_{\rm g}^{\mu\nu;\lambda\kappa}(q,q')^{(0)}$ *is therefore gauge independent.*

Next check $G_{\rm m}^{\mu\nu}(k,p)^{(0)}$: $Y^{\mu\nu}(q)$ in (2.30) is given by

$$Y^{\mu\nu}(q) \equiv \frac{1}{2}\Gamma^{\mu\nu\lambda}q_\lambda + mg^{\mu\nu} \ , \tag{2.37}$$

with

$$\Gamma^{\mu\nu\lambda} \equiv \frac{1}{2}(\gamma^\mu g^{\nu\lambda} + \gamma^\nu g^{\mu\lambda}) - \gamma^\lambda g^{\mu\nu} \ . \tag{2.38}$$

According to our assumption, (2.10) to (2.11), there is no gauge dependence in the electron sector: $Y^{\mu\nu}$ *is gauge invariant so is* $G_{\rm m}^{\mu\nu}(k,p)^{(0)}$.

The 1PI part of $G_{\rm m}^{\mu\nu;\lambda}(q;k,p)^{(0)}$, $Z^{\mu\nu\lambda}(q)$ in (2.30), is expressed, in terms of (2.38), by

$$Z^{\mu\nu\lambda}(q) \equiv \Gamma^{\mu\nu\rho}d_\rho{}^\lambda(q) \ . \tag{2.39}$$

As is seen from Fig.2 reducible graphs take part in

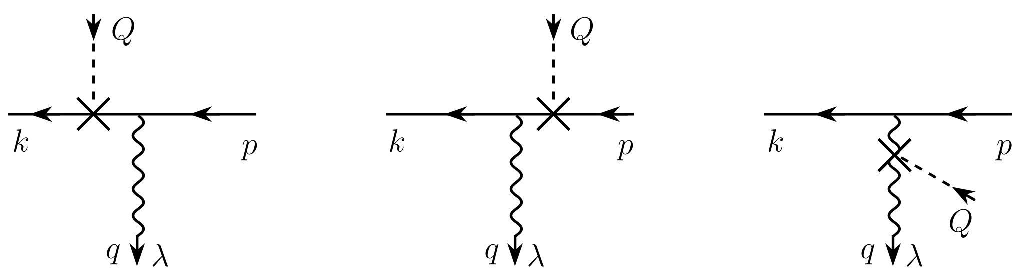


Fig.2

giving totally

$$\begin{aligned} &\frac{\not k - m}{i}G_{\rm m}^{\mu\nu;\lambda}(q;k,p)^{(0)}\frac{\not p - m}{i} \\ &= \frac{ie}{q^2}\left[Y^{\mu\nu}(k+p-q)\frac{1}{\not p - \not q - m}\gamma^\lambda + \gamma^\lambda\frac{1}{\not k + \not q - m}Y^{\mu\nu}(k+p+q) - \Gamma^{\mu\nu\lambda}\right] \\ &-\frac{ie(1-\alpha)q^\lambda}{(q^2)^2}\left[Y^{\mu\nu}(k+p-q)\frac{1}{\not p - \not q - m}(\not p - m) - (\not k - m)\frac{1}{\not k + \not q - m}Y^{\mu\nu}(k+p+q)\right] , \end{aligned} \tag{2.40}$$

whose α-dependent terms vanish due to the factor q^λ or $(\not{k}-m)$ as well as $(\not{p}-m)$.

Finally $G_{\rm g}^{\mu\nu;\lambda}(q;k,p)^{(0)}$ is also gauge invariant:

$$\begin{aligned}
&\frac{\not{k}-m}{i}G_{\rm g}^{\mu\nu;\lambda}(q;k,p)^{(0)}\frac{\not{p}-m}{i} \\
&=\frac{-ie}{q^2(k-p)^2}\gamma^\rho\widetilde{X}^{\mu\nu\lambda}{}_\rho(q,k-p) \\
&+\frac{ie}{q^2(k-p)^2}\left\{q^\lambda\gamma^\rho X^{\mu\nu}{}_\rho(q,k-p)+(\not{k}-\not{p})\left[X^{\mu\nu\lambda}(k-p,q)-\alpha q^\lambda g^{\mu\nu}\right]\right\}\ ;
\end{aligned} \tag{2.41}$$

since the second term in the right hand side again vanishes because of q^λ and $(\not{k}-m)$ as well as $(\not{p}-m)$. (Recall that $\widetilde{X}^{\mu\nu\lambda\rho}(q,q')$ is gauge invariant.)

All the tree graphs coupled to physical states are thus gauge independent.

2.3 One-loop Calculation

$G_{\rm g}^{\mu\nu;\lambda\kappa}(q,q')^{(1)}$: contributions are from the graphs, p_1 and p_2.

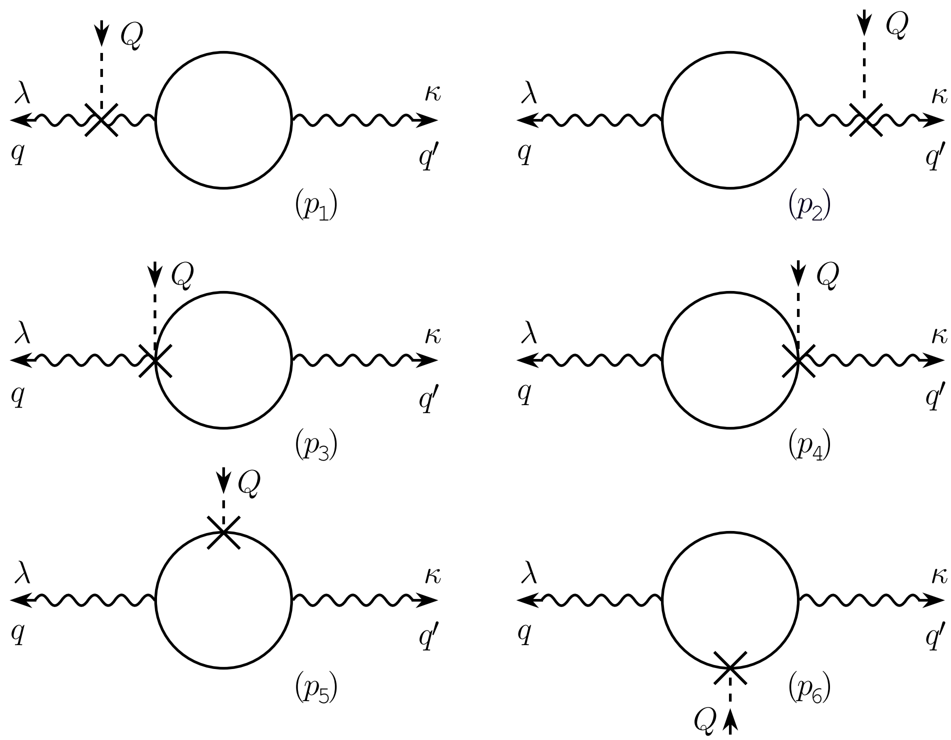


*Fig.*3

$$\begin{aligned}
&G_{\rm g}^{\mu\nu;\lambda\kappa}(q,q')^{(1)} \\
&=\frac{-i}{q^2}X^{\mu\nu\lambda\rho}(q,q')\frac{-i}{q'^2}\Pi_{\rho\sigma}(q')\frac{-i}{q'^2}d^{\sigma\kappa}(q')+\frac{-i}{q^2}d^{\lambda\rho}(q)\Pi_{\rho\sigma}(q)\frac{-i}{q^2}X^{\mu\nu\sigma\kappa}(q,q')\frac{-i}{q'^2}\ ,
\end{aligned} \tag{2.42}$$

where $\mathit{\Pi}^{\rho\sigma}(q)$ is the vacuum polarization,

$$\begin{aligned}
&\mathit{\Pi}^{\rho\sigma}(q) \equiv -e^2\int\frac{d^n l}{(2\pi)^n}tr\left[\gamma^\rho\frac{1}{\not l+\not q-m}\gamma^\sigma\frac{1}{\not l-m}\right] \ ; \quad \mathit{\Pi}^{\rho\sigma}(q) = (q^2 g^{\rho\sigma}-q^\rho q^\sigma)\mathit{\Pi}(q^2) \ , \\
&\mathit{\Pi}(q) \equiv -ie^2\frac{2\,\mathrm{tr}\mathbf{1}}{(4\pi)^2}\mathit{\Gamma}(2-\frac{n}{2})\int_0^1 dx\ x(1-x)\left(\frac{m^2-x(1-x)q^2}{4\pi}\right)^{\frac{n}{2}-2} \quad ;
\end{aligned} \tag{2.43}$$

obeying the transversal condition:

$$q_\rho\mathit{\Pi}^{\rho\sigma}(q) = 0 \ . \tag{2.44}$$

As was mentioned before, gauge dependent parts, in $X^{\mu\nu\lambda\kappa}$ (2.31) and $d^{\lambda\kappa}$ (2.34), are proportional to q^λ and/or q'^κ, then they vanish on account of the physical photon condition (2.28) when the momentum is external or of the transverseness of the vacuum polarization (2.44) when it is internal.

$G_{\mathrm{m}}^{\mu\nu;\lambda\kappa}(q,q')^{(1)}$: graphs are given in $p_3 \sim p_6$.

$$G_{\mathrm{m}}^{\mu\nu;\lambda\kappa}(q,q')^{(1)} \equiv \frac{-i}{q^2}d^\lambda{}_\rho(q)\mathit{\Pi}^{\mu\nu;\rho\sigma}(q,q')d_\sigma{}^\kappa(q')\frac{-i}{q'^2} \ , \tag{2.45}$$

where

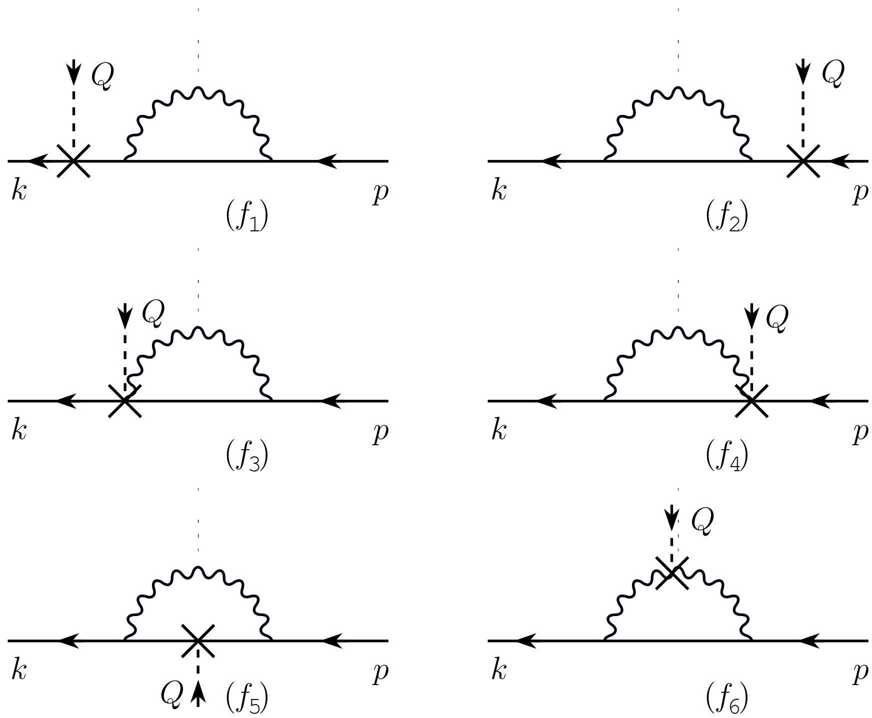
$$\begin{aligned}
\mathit{\Pi}^{\mu\nu;\lambda\kappa}(q,q') \equiv ie^2\int\frac{d^n l}{(2\pi)^n}\mathrm{tr}\Bigg[&\mathit{\Gamma}^{\mu\nu\lambda}\frac{1}{\not l-\not q'/2-m}\gamma^\kappa\frac{1}{\not l+\not q'/2-m} \\
&+\gamma^\lambda\frac{1}{\not l+\not q/2-m}\mathit{\Gamma}^{\mu\nu\kappa}\frac{1}{\not l-\not q/2-m} \\
-\gamma^\lambda\frac{1}{\not l+\not q/2+\not q'/2-m}&Y^{\mu\nu}(2l)\frac{1}{\not l-\not q/2-\not q'/2-m}\gamma^\kappa\frac{1}{\not l-\not q/2+\not q'/2-m} \\
-\gamma^\lambda\frac{1}{\not l+\not q/2-\not q'/2-m}&\gamma^\kappa\frac{1}{\not l+\not q/2+\not q'/2-m}Y^{\mu\nu}(2l)\frac{1}{\not l-\not q/2-\not q'/2-m}\Bigg] \ ,
\end{aligned} \tag{2.46}$$

which is free from gauge dependence, reflecting the gauge independence of $\mathit{\Theta}_{\mathrm{m}}^{\mu\nu}$. The only dangerous part is therefore the gauge term in the propagators, $d^\lambda{}_\rho(q), d_\sigma{}^\kappa(q')$. However, $\mathit{\Pi}^{\mu\nu;\lambda\kappa}(q,q')$ has a remarkable property,

$$q_\lambda\mathit{\Pi}^{\mu\nu;\lambda\kappa}(q,q') = q'_\kappa\mathit{\Pi}^{\mu\nu;\lambda\kappa}(q,q') = 0 \ , \tag{2.47}$$

with which there remains no gauge dependence in (2.45). (Or the physical state condition for photon (2.28) also wipes out the gauge dependence in this case.) The relation (2.47) is easily recognized by means of a simple and straightforward manipulation such that $\not q = (\not l+\not q/2-m)-(\not l-\not q/2-m)$ or the Ward-Takahashi relation discussed below.

$G^{\mu\nu}_{\rm m}(k,p)^{(1)}$: graphs are $f_1 \sim f_5$.



$Fig.4$

$$
\begin{aligned}
&\frac{\not k - m}{i} G^{\mu\nu}_{\rm m}(k,p)^{(1)} \frac{\not p - m}{i} \\
&= -ie^2 \int \frac{d^n l}{(2\pi)^n} \Big\{ \frac{1}{l^2} \Big[-\Gamma^{\mu\nu\rho} \frac{1}{\not p - \not l - m} \gamma_\rho - \gamma_\rho \frac{1}{\not k - \not l - m} \Gamma^{\mu\nu\rho} \\
&\quad + Y^{\mu\nu}(k+p) \frac{1}{\not p - m} \gamma^\rho \frac{1}{\not p - \not l - m} \gamma_\rho + \gamma_\rho \frac{1}{\not k - \not l - m} \gamma^\rho \frac{1}{\not p - \not l - m} Y^{\mu\nu}(k+p) \\
&\quad + \gamma^\rho \frac{1}{\not k - \not l - m} Y^{\mu\nu}(k+p-2l) \frac{1}{\not p - \not l - m} \gamma_\rho \Big] \\
&\quad - \frac{(1-\alpha)}{(l^2)^2} \Big[(\not k - m) \frac{1}{\not k - \not l - m} Y^{\mu\nu}(k+p-2l) \frac{1}{\not p - \not l - m} (\not p - m) - Y^{\mu\nu}(k+p) \Big] \Big\} \ .
\end{aligned}
\tag{2.48}
$$

The gauge dependence appears only in the last line whose first term, however, vanishes by the on-shell condition of electron (2.29) so does the second term owing to the property of the dimensional regularization:

$$
\int \frac{d^n l}{(2\pi)^n} \frac{1}{(l^2)^N} = 0 \ ; \qquad N\text{: integer.} \tag{2.49}
$$

$G_{\rm g}^{\mu\nu}(k,p)^{(1)}$: there contributes only one graph, f_6.

$$
\begin{aligned}
&\frac{\not k - m}{i} G_{\rm g}^{\mu\nu}(k,p)^{(1)} \frac{\not p - m}{i} \\
&= ie^2 \int \frac{d^n l}{(2\pi)^n} \frac{1}{L^2 (L-Q)^2} X^{\mu\nu\lambda\kappa}(Q-L, L)\gamma_\lambda \frac{1}{\not p + \not L - m}\gamma_\kappa \\
&= ie^2 \int \frac{d^n l}{(2\pi)^n} \frac{1}{L^2 (L-Q)^2} \Big[\widetilde{X}^{\mu\nu\lambda\kappa}(Q-L, L)\gamma_\lambda \frac{1}{\not p + \not L - m}\gamma_\kappa \\
&-X^{\mu\nu\kappa}(Q-L, L)(\not k - m)\frac{1}{\not p + \not L - m}\gamma_\kappa \\
&+X^{\mu\nu\lambda}(L, Q-L)\gamma_\lambda \frac{1}{\not p + \not L - m}(\not p - m) \\
&+\alpha g^{\mu\nu}\left\{ \frac{\not k + \not p}{2} - m - (\not k - m)\frac{1}{\not p + \not L - m}(\not p - m) \right\} \Big] \ ,
\end{aligned}
\tag{2.50}
$$

where

$$
L \equiv l + \frac{Q}{2} \ , \qquad Q \equiv k - p \ . \tag{2.51}
$$

In the final expression, terms from the second to the last line are gauge dependent but the on-shell condition of electron wipes them out.

Note that the important relations for obtaining the gauge independent results are (2.44) and (2.47). We must discuss $G_{\rm a}^{\mu\nu;\lambda}(q;k,p)^{(1)}$ to complete the one loop calculation. The scenario, however, can be realized and furthermore generalized to any order with the aid of the Ward-Takahashi relation.

2.4 General Proof for Gauge Independence

We here show that gauge independence of the energy-momentum tensor holds in any order of the loop expansion. However as is seen in the following, *all photon lines can be treated as external* and *there needs only for considering the tree and the one loop graphs of electron.* To grasp this we should note that

- (a) All gauge dependent terms, in $\Theta_{\rm g}^{\mu\nu}$ (, in view of $X^{\mu\nu\lambda\kappa}(q,q')$ (2.31),) and the propagator (2.34), possess a momentum contractible with a vertex or a photon wave function. The latter vanishes trivially on account of the physical photon condition (2.28) so that we concentrate on the former. To check gauge independence is therefore to check the consequence of the photon momentum contraction to the vertex. The procedure is exactly the same as studying gauge independence of the S-matrix; in other words, checking the cancellation of gauge terms in the photon propagator [17, 2].
- (b) On the other hand, the insertion of $\Theta_{\rm m}^{\mu\nu}$ forms new types of graphs. However $\Theta_{\rm m}^{\mu\nu}$ itself is gauge invariant so that gauge dependence lies in the photon propagator. Checking gauge independence is again realized by the same manipulation of momentum as in (a) to those new graphs.
- (c) Because of the fact (a) any internal photon line can be cut out. Graphs that must be taken into account are therefore the tree and the one loop graphs of electron.

The program is carried out by means of the Ward-Takahashi relation (WT) derived by applying the BRS transformation to the generating functional W (2.26) [16]:

$$
\langle 0| \left[Q_B, T^* \exp\left[i\int d^4x \left\{ J_\mu A^\mu + JB + \overline{\eta}\psi + \overline{\psi}\eta + \tau_{\mu\nu}^{\rm m}\Theta_{\rm m}^{\mu\nu} + \tau_{\mu\nu}^{\rm g}\Theta_{\rm g}^{\mu\nu} \right\}\right]\right] |0\rangle = 0 \ , \tag{2.52}
$$

which becomes in view of (1.13) and (1.9)

$$\left[e\left(\overline{\eta}\frac{\delta}{\delta\overline{\eta}}-\eta\frac{\delta}{\delta\eta}\right)+\Box\frac{\delta}{i\delta J}+(\delta^\mu_\rho\delta^\nu_\sigma+\delta^\mu_\sigma\delta^\nu_\rho-g^{\mu\nu}g_{\rho\sigma})\partial^\rho\Bigl(\tau^{\rm g}_{\mu\nu}\partial^\sigma\frac{\delta}{i\delta J}\Bigr)\right]W[\boldsymbol{J},\boldsymbol{\eta},\boldsymbol{\tau}]+i\partial^\rho J_\rho=0\ . \quad (2.53)$$

In order to simplify a discussion we further introduce the generating functional of photon amputated Green's functions,

$$\Gamma[\boldsymbol{A};\boldsymbol{\eta},\boldsymbol{\tau}]\equiv W[\boldsymbol{J},\boldsymbol{\eta},\boldsymbol{\tau}]-\int d^4xJ^\mu(x)\mathcal{A}_\mu(x)-\int d^4xJ(x)\mathcal{B}(x)\ , \quad (2.54)$$

where

$$\frac{\delta W[\boldsymbol{J},\boldsymbol{\eta},\boldsymbol{\tau}]}{\delta J^\mu}\equiv\mathcal{A}_\mu\ ,\qquad \frac{\delta W[\boldsymbol{J},\boldsymbol{\eta},\boldsymbol{\tau}]}{\delta J}\equiv\mathcal{B}\ . \quad (2.55)$$

Γ also obeys WT obtaining from (2.53):

$$\begin{aligned}&\left[e\left(\overline{\eta}\frac{\delta}{\delta\overline{\eta}}-\eta\frac{\delta}{\delta\eta}\right)-i\partial^\rho\frac{\delta}{\delta\mathcal{A}^\rho}\right]\Gamma[\boldsymbol{A};\boldsymbol{\eta},\boldsymbol{\tau}]-i\Box\mathcal{B}\\ &\qquad\qquad -i(\delta^\mu_\rho\delta^\nu_\sigma+\delta^\mu_\sigma\delta^\nu_\rho-g^{\mu\nu}g_{\rho\sigma})\partial^\rho\Bigl(\tau^{\rm g}_{\mu\nu}\partial^\sigma\mathcal{B}\Bigr)=0\ .\end{aligned} \quad (2.56)$$

Owing to the above discussion, we need only the tree $\Gamma^{(0)}$ and the one loop $\Gamma^{(1)}$.

$\Theta^{\mu\nu}_{\rm g}$**-inserted part** :a typical subdiagram is seen in Fig.5.

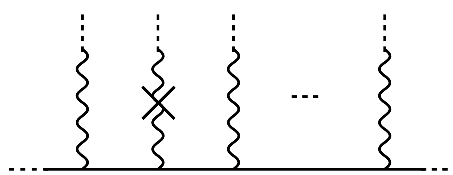


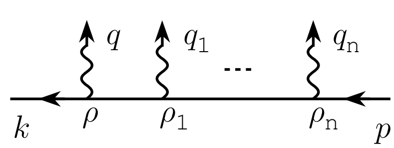
*Fig.*5

From (2.56),

$$\left[e\left(\overline{\eta}\frac{\delta}{\delta\overline{\eta}}-\eta\frac{\delta}{\delta\eta}\right)-i\partial^\rho\frac{\delta}{\delta\mathcal{A}^\rho}\right]\Gamma[\boldsymbol{A};\boldsymbol{\eta},\boldsymbol{\tau}]\Bigg|_{\mathcal{B}=\boldsymbol{\tau}=0}=0\ . \quad (2.57)$$

Write the tree Green's functions of electron as

$$\begin{aligned}&i(-e)^{n+1}I^{\rho\rho_1\cdots\rho_n}(q,q_1,\cdots,q_n;k,p)\\ &\equiv\int d^4yd^4z\prod_{j=1}^n d^4x_j\exp[iky-ipz+i\sum_{j=1}^n q_jx_j]\\ &\times\frac{\delta^2}{\delta\overline{\eta}(y)\delta\eta(z)}\frac{i\delta^{n+1}\Gamma^{(0)}[\boldsymbol{A};\boldsymbol{\eta},\boldsymbol{\tau}]}{\delta\mathcal{A}_\rho(0)\delta\mathcal{A}_{\rho_1}(x_1)\cdots\delta\mathcal{A}_{\rho_n}(x_n)}\Bigg|_{\boldsymbol{A}=\boldsymbol{\eta}=\boldsymbol{\tau}=0}\\ &\equiv\sum_{\substack{permutation\\ \{\binom{q}{\rho},\binom{q_1}{\rho_1},\cdots,\binom{q_n}{\rho_n}\}}}\end{aligned} \quad (2.58)$$



,

with $q = p - k - \sum q_j$. Therefore WT reads

$$q_\rho I^{\rho\rho_1\cdots\rho_n}(q, q_1, \cdots, q_n; k, p) = -I^{\rho_1\cdots\rho_n}(q_1, \cdots, q_n; k+q, p) + I^{\rho_1\cdots\rho_n}(q_1, \cdots, q_n; k, p-q) \quad (2.59)$$

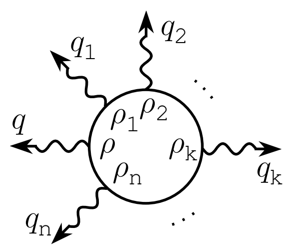
whose left hand side stands for the contribution from the gauge dependent part in the photon propagator. The LSZ amplitude is obtained by sandwiching (2.59) with $\overline{u}(k,s)(\not{k} - m)$ and $(\not{p} - m)u(p, s')$. The gauge dependent part is therefore becomes finally

$$\overline{u}(k,s)(\not{k} - m)q_\rho I^{\rho\rho_1\cdots\rho_n}(q, q_1, \cdots, q_n; k, p)(\not{p} - m)u(p, s') = 0 \; ; \quad (2.60)$$

since the right hand side of (2.59) cannot escape the cancellation because of the momentum shift $k+q$ or $p-q$.

For the one loop subgraphs define a Green's function,

$$\begin{aligned}
&(-e)^{n+1}\Pi^{\rho\rho_1\cdots\rho_n}(q, q_1, \cdots, q_n) \\
&\equiv \int \prod_{j=1}^{n} d^4x_j \exp[i\sum_{j=1}^{n} q_j x_j] \left.\frac{i\delta^{n+1}\Gamma^{(1)}[\boldsymbol{A};\boldsymbol{\eta},\boldsymbol{\tau}]}{\delta\mathcal{A}_\rho(0)\delta\mathcal{A}_{\rho_1}(x_1)\cdots\delta\mathcal{A}_{\rho_n}(x_n)}\right|_{\boldsymbol{A}=\boldsymbol{\eta}=\boldsymbol{\tau}=0} \\
&\equiv \sum_{\substack{permutation \\ \{\binom{q_1}{\rho_1},\cdots,\binom{q_n}{\rho_n}\}}}
\end{aligned} \quad (2.61)$$



,

where $q = -\sum q_j$. WT reads

$$q_\rho \Pi^{\rho\rho_1\cdots\rho_n}(q, q_1, \cdots, q_n) = 0 \quad (2.62)$$

which tells us that gauge dependence also disappears.

$\Theta_{\mathrm{m}}^{\mu\nu}$-inserted part : there are two type of typical subdiagrams with a $\Theta_{\mathrm{m}}^{\mu\nu}$ insertion in Fig.8.

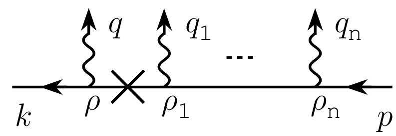
*Fig.*8

In this case WT reads

$$\left[e\left(\overline{\eta}\frac{\delta}{\delta\overline{\eta}} - \eta\frac{\delta}{\delta\eta}\right) - i\partial^\rho \frac{\delta}{\delta\mathcal{A}^\rho}\right]\frac{\delta\Gamma[\boldsymbol{A};\boldsymbol{\eta},\boldsymbol{\tau}]}{\delta\tau_{\mu\nu}^{\mathrm{m}}} = 0 \; . \quad (2.63)$$

Define $\Theta_{\rm m}^{\mu\nu}$ inserted Green's functions in the tree order:

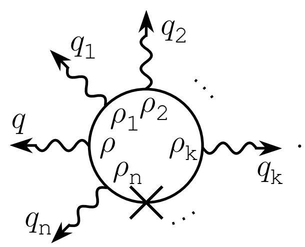
$$
\begin{aligned}
&(-e)^{n+1} I^{\mu\nu;\rho\rho_1\cdots\rho_n}(q,q_1,\cdots,q_n;k,p) \\
&\equiv \int d^4y d^4z d^4x \prod_{j=1}^{n} d^4x_j \exp[iky - ipz + iqx + i\sum_{j=1}^{n} q_j x_j] \\
&\times \frac{\delta^2}{\delta\overline{\eta}(y)\delta\eta(z)} \frac{\delta}{\delta\tau_{\mu\nu}^{\rm m}(0)} \frac{\delta^{n+1}\Gamma^{(0)}[\boldsymbol{A};\boldsymbol{\eta},\boldsymbol{\tau}]}{\delta\mathcal{A}_\rho(x)\delta\mathcal{A}_{\rho_1}(x_1)\cdots\delta\mathcal{A}_{\rho_n}(x_n)}\Bigg|_{\boldsymbol{A}=\boldsymbol{\eta}=\boldsymbol{\tau}=0} \qquad (2.64)\\
&\equiv \sum_{\substack{\textit{permutation}\\ \{\binom{q}{\rho},\binom{q_1}{\rho_1},\cdots,\binom{q_n}{\rho_n}\}}} \;\; \sum_{\substack{\textit{all possible}\\ \textit{insertions of } \Theta_{\rm m}^{\mu\nu}}}
\end{aligned}
$$



,

and in the one loop:

$$
\begin{aligned}
&-i(-e)^{n+1}\Pi^{\mu\nu;\rho\rho_1\cdots\rho_n}(q,q_1,\cdots,q_n) \\
&\equiv \int d^4x \prod_{j=1}^{n} d^4x_j \exp[iqx + i\sum_{j=1}^{n} q_j x_j] \frac{\delta}{\delta\tau_{\mu\nu}^{\rm m}(0)} \frac{\delta^{n+1}\Gamma^{(1)}[\boldsymbol{A};\boldsymbol{\eta},\boldsymbol{\tau}]}{\delta\mathcal{A}_\rho(x)\delta\mathcal{A}_{\rho_1}(x_1)\cdots\delta\mathcal{A}_{\rho_n}(x_n)}\Bigg|_{\boldsymbol{A}=\boldsymbol{\eta}=\boldsymbol{\tau}=0} \qquad (2.65)\\
&\equiv \sum_{\substack{\textit{permutation}\\ \{\binom{q_1}{\rho_1},\cdots,\binom{q_n}{\rho_n}\}}} \;\; \sum_{\substack{\textit{all possible}\\ \textit{insertions of } \Theta_{\rm m}^{\mu\nu}}}
\end{aligned}
$$



.

WT for $I^{\mu\nu}$'s and $\Pi^{\mu\nu}$'s are given as the same as (2.59) and (2.62);

$$
\begin{aligned}
&q_\rho I^{\mu\nu;\rho\rho_1\cdots\rho_n}(q,q_1,\cdots,q_n;k,p) \\
&= -I^{\mu\nu;\rho_1\cdots\rho_n}(q_1,\cdots,q_n;k+q,p) + I^{\mu\nu;\rho_1\cdots\rho_n}(q_1,\cdots,q_n;k,p-q)\ ,
\end{aligned}
\tag{2.66}
$$

$$
q_\rho \Pi^{\mu\nu;\rho\rho_1\cdots\rho_n}(q,q_1,\cdots,q_n) = 0\ . \tag{2.67}
$$

(2.67) is considered as a generalization of the previous relation (2.47). From (2.66) and (2.67) gauge dependence is wiped out again by a similar manner as in the $\Theta_{\rm m}^{\mu\nu}$-inserted case.

Needless to say, $G_{\rm g}^{\mu\nu;\lambda}(q;k,p)^{(1)}$ belong to the case (2.58) and $G_{\rm m}^{\mu\nu;\lambda}(q;k,p)^{(1)}$ to (2.64).We have now convinced ourselves that the energy-momentum tensor (2.23) is gauge independent under the LSZ-formalism, which, in other words, guarantees the passage from (2.10) to (2.11). The asymptotic electron field is therefore considered as gauge invariant but the relationship to the interpolating field is still unclear.

3 Gauge Invariant Approaches

According to discussions in the foregoing sections, the LSZ asymptotic states is gauge invariant with the use of the physical state conditions (2.28) and (2.29). However, it is preferable to introduce gauge invariant operators for photon and electron, which would guarantee gauge independence more directly.

To investigate that let us recall the gauge invariant quantities in (classical) electrodynamics: the minimal coupling term,

$$\overline{\psi}(x) i\gamma^{\mu}(\partial_{\mu} - ieA_{\mu}(x))\psi(x) \ , \tag{3.1}$$

and the field strength tensor $F_{\mu\nu}(x)$.

The gauge transformation is expressed as

$$\begin{aligned} &A_{\mu}(x) \mapsto A_{\mu}(x) + \partial_{\mu}\chi(x) \ , \\ &\psi(x) \mapsto e^{ie\chi(x)}\psi(x) \ , \\ &\overline{\psi}(x) \mapsto \overline{\psi}(x) e^{-ie\chi(x)} \ . \end{aligned} \tag{3.2}$$

In terms of the components, the photon part of (3.2) reads as

$$\begin{aligned} &A_0(x) \mapsto A_0(x) + \dot{\chi}(x) \ , \\ &\boldsymbol{A}(x) \mapsto \boldsymbol{A}(x) - \boldsymbol{\nabla}\chi(x) \ . \end{aligned} \tag{3.3}$$

Now we decompose the vector potential $\boldsymbol{A}(x)$ into

$$\boldsymbol{A}(x) = \boldsymbol{A}_{\mathrm{T}}(x) + \boldsymbol{A}_{\mathrm{L}}(x) \ , \tag{3.4}$$

where $\boldsymbol{A}_{\mathrm{T}}(x)(\boldsymbol{A}_{\mathrm{L}}(x))$ denotes the transverse (longitudinal) component with respect to the derivative $\boldsymbol{\nabla}$; thus

$$\begin{aligned} &\boldsymbol{\nabla} \cdot \boldsymbol{A}_{\mathrm{T}}(x) = 0 \ , \\ &\boldsymbol{\nabla} \times \boldsymbol{A}_{\mathrm{L}}(x) = 0 \ . \end{aligned} \tag{3.5}$$

In view of (3.2), we then obtain the transformation rule:

$$\begin{aligned} &\boldsymbol{A}_{\mathrm{T}}(x) \mapsto \boldsymbol{A}_{\mathrm{T}}(x) \ , \\ &\boldsymbol{A}_{\mathrm{L}}(x) \mapsto \boldsymbol{A}_{\mathrm{L}}(x) - \boldsymbol{\nabla}\chi(x) \ . \end{aligned} \tag{3.6}$$

From this we recognize that *the transverse part,* $\mathbf{A}_{\mathrm{T}}(x)$,*is gauge invariant* [5]. The vector $\boldsymbol{\nabla}$ sets up a reference axis along which gauge invariant quantities can be constructed. In order to find other invariant quantities, let us go back to (3.1). First it should be noticed that

$$\begin{aligned} &\psi^{\mathrm{C}}_{\mathrm{inv}}(x) \equiv \exp\Big[ie\int^{\boldsymbol{x}} d\boldsymbol{z} \cdot \boldsymbol{A}_{\mathrm{L}}(x^0, \boldsymbol{z})\Big]\psi(x) \ , \\ &\overline{\psi}^{\mathrm{C}}_{\mathrm{inv}}(x) \equiv \overline{\psi}(x) \exp\Big[-ie\int^{\boldsymbol{x}} d\boldsymbol{z} \cdot \boldsymbol{A}_{\mathrm{L}}(x^0, \boldsymbol{z})\Big] \ , \end{aligned} \tag{3.7}$$

are gauge invariant under (3.3) and (3.6), path-independent owing to (3.5), (hence the beginning point of the integral can be arbitrary), and *in fact local.* This is essentially the Dirac's physical electron [3, 7]; since

$$\boldsymbol{A}_{\mathrm{L}}(x) = \boldsymbol{\nabla}\frac{\boldsymbol{\nabla} \cdot \boldsymbol{A}}{\boldsymbol{\nabla}^2}(x) \ , \tag{3.8}$$

where

$$\frac{f}{\boldsymbol{\nabla}^2}(x) \equiv -\frac{1}{4\pi}\int_{-\infty}^{\infty} d^3\boldsymbol{y} \ \frac{f(x_0, \boldsymbol{y})}{|\boldsymbol{x} - \boldsymbol{y}|} \ , \tag{3.9}$$

so that (3.7) becomes

$$\begin{aligned} &\psi^{\mathrm{C}}_{\mathrm{inv}}(x) = \exp\Big[ie\frac{\boldsymbol{\nabla} \cdot \boldsymbol{A}}{\boldsymbol{\nabla}^2}(x)\Big] \ \psi(x) \ , \\ &\overline{\psi}^{\mathrm{C}}_{\mathrm{inv}}(x) = \overline{\psi}(x) \exp\Big[-ie\frac{\boldsymbol{\nabla} \cdot \boldsymbol{A}}{\boldsymbol{\nabla}^2}(x)\Big] \ , \end{aligned} \tag{3.10}$$

which is the Dirac's electron.

The minimal coupling term (3.1) becomes

$$\overline{\psi}^{\mathrm{C}}_{\mathrm{inv}}(x)i\left[\gamma^0\left\{\partial_0 - ie\Big(A_0(x) + \int^{\boldsymbol{x}} d\boldsymbol{z}\cdot\dot{\boldsymbol{A}}_{\mathrm{L}}(x^0,\boldsymbol{z})\Big)\right\} + \boldsymbol{\gamma}\cdot\Big(\boldsymbol{\nabla} + ie\boldsymbol{A}_{\mathrm{T}}(x)\Big)\right]\psi^{\mathrm{C}}_{\mathrm{inv}}(x)\ , \tag{3.11}$$

leading to the gauge invariant potential,

$$A^{\mathrm{C}}_{\mu}(x) \equiv \left(A^{\mathrm{C}}_0(x), -\boldsymbol{A}^{\mathrm{C}}(x)\right)\ , \tag{3.12}$$

with $\boldsymbol{A}^{\mathrm{C}}(x) \equiv \boldsymbol{A}_{\mathrm{T}}(x)$, and

$$A^{\mathrm{C}}_0(x) \equiv A_0(x) + \int^{\boldsymbol{x}} d\boldsymbol{z}\cdot\dot{\boldsymbol{A}}_{\mathrm{L}}(x^0,\boldsymbol{z})\ . \tag{3.13}$$

Apparently

$$\boldsymbol{\nabla}\cdot\boldsymbol{A}^{\mathrm{C}}(x) = 0\ . \tag{3.14}$$

In view of (3.14) this is nothing but the Coulomb gauge. From this lesson, it should be recognized that *to set up gauge invariant operators is nothing but to fix the gauge,* whose result also can be seen in a covariant form by Steinmann [4]. We should use the terminology "BRS invariance" here instead of gauge invariance since we now move into quantum theory. The starting Lagrangian is the Nakanishi-Lautrup one (1.7) and the BRS transformation is given by (1.9). (The Faddeev-Popov ghosts are irrelevant all the time in QED.) The BRS invariant fermion fields are thus defined by

$$\begin{aligned}\Psi(x) &\equiv \psi^{\phi}_{\mathrm{inv}}(x) \equiv \exp\left[-ie\int d^4y\,\phi^{\mu}(x-y)A_{\mu}(y)\right]\psi(x)\ ,\\ \overline{\Psi}(x) &\equiv \overline{\psi}^{\phi}_{\mathrm{inv}}(x) \equiv \overline{\psi}(x)\exp\left[ie\int d^4y\,\phi^{\mu}(x-y)A_{\mu}(y)\right]\ ,\end{aligned} \tag{3.15}$$

with a real distribution $\phi^{\mu}(x)$ satisfying

$$\partial_{\mu}\phi^{\mu}(x) = \delta^4(x)\ . \tag{3.16}$$

The minimal coupling term becomes

$$\overline{\Psi}(x)\left[i\partial\!\!\!/ - m + e\gamma^{\nu}\int d^4y\,\phi^{\mu}(x-y)F_{\mu\nu}(y)\right]\Psi(x) \tag{3.17}$$

so that the BRS invariant potential in the Steinmann's approach is

$$\begin{aligned}A^{\phi}_{\mu}(x) &\equiv -\int d^4y\,\phi^{\nu}(x-y)F_{\mu\nu}(y)\\ &= A_{\mu}(x) - \partial^x_{\mu}\int d^4y\,\phi^{\nu}(x-y)A_{\nu}(y)\\ &= \int\frac{d^4q}{(2\pi)^4}\mathrm{e}^{-iqx}(\delta^{\lambda}_{\mu} + iq_{\mu}\phi^{\lambda}(q))\int d^4y\,\mathrm{e}^{iqy}A_{\lambda}(y)\ ,\end{aligned} \tag{3.18}$$

where use has been made of the Fourier transformation

$$\phi^{\mu}(x) = \int\frac{d^4q}{(2\pi)^4}\phi^{\mu}(q)\mathrm{e}^{-iqx}\ . \tag{3.19}$$

According to the second expression in (3.18), it can be regarded that A_μ has been decomposed into the gauge invariant part, A^ϕ_μ, and the variant part, $\overline{A}_\mu(x)$;

$$\begin{aligned} A_\mu(x) &= A^\phi_\mu(x) + \overline{A}_\mu(x) \ ; \\ \overline{A}_\mu(x) &\equiv \partial^x_\mu \int d^4y\, \phi^\nu(x-y) A_\nu(y) \ , \end{aligned} \tag{3.20}$$

which corresponds to (3.4). In this case, the reference vector is of course ϕ^μ.

(3.16) implies

$$q_\mu \phi^\mu(q) = i \ , \tag{3.21}$$

yielding to

$$q_\mu \Big(\delta^\mu_\nu + i q_\nu \phi^\mu(q)\Big) = 0 \ ; \quad \phi^\mu(-q) = -\phi^\mu(q) \ , \tag{3.22}$$

which furthermore leads to a projection property:

$$\Big(\delta^\mu_\nu + i q_\nu \phi^\mu(q)\Big)\Big(\delta^\nu_\lambda + i q_\lambda \phi^\nu(q)\Big) = \Big(\delta^\mu_\lambda + i q_\lambda \phi^\mu(q)\Big) \ . \tag{3.23}$$

If

$$\phi^\mu(x) = \Big(0, \frac{\boldsymbol{x}}{4\pi|\boldsymbol{x}|^3}\delta(x_0)\Big) \ , \tag{3.24}$$

(3.15) becomes the Dirac's electron (3.10) and (3.18) becomes (3.12). Likewise, if the support of ϕ^μ lies in a space-like region BRS invariant operators, (3.15) and (3.18), are well-defined by all means. On the contrary, if ϕ^μ's support includes a time-like region, the expressions for Ψ's lose its meaning by themselves because of the non-commutativity of A's and ψ. However, even in this case perturbation can lay down the definition: for example, considering the quantity,

$$\langle 0|\, T^* A^{\phi\lambda_1}(x_1)\cdots A^{\phi\lambda_n}(x_n)\Psi(y_1)\cdots\Psi(y_m)\overline{\Psi}(z_1)\cdots\overline{\Psi}(z_m)\,|0\rangle \ , \tag{3.25}$$

in terms of perturbation, imparts a meaning to those BRS invariant operators. (3.25) can, however, be more simplified, in view of (3.18), such that

$$\begin{aligned} &\langle 0|\, T^* A^{\lambda_1}(x_1)\cdots A^{\lambda_n}(x_n)\Psi(y_1)\cdots\Psi(y_m)\overline{\Psi}(z_1)\cdots\overline{\Psi}(z_m)\,|0\rangle \\ &= \langle 0|\, T^* A^{\lambda_1}(x_1)\cdots A^{\lambda_n}(x_n)\psi(y_1)\cdots\psi(y_m)\overline{\psi}(z_1)\cdots\overline{\psi}(z_m) \\ &\times \exp\Big[-ie\int d^4s\, \phi^\rho(y_1-s)A_\rho(s)\Big]\cdots\exp\Big[ie\int d^4s\, \phi^\rho(z_m-s)A_\rho(s)\Big]\,|0\rangle \ . \end{aligned} \tag{3.26}$$

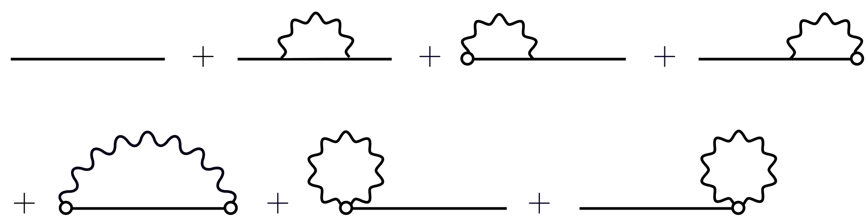
Therefore the effect of the physical electron is solely found in the additional vertex, ϕ^μ-vertex as in Fig.11.

*Fig.*11

Furthermore by noting that $\varPsi$ simply turns out to be ψ under the loops, tree graphs of electron are only relevant. Especially for the two-point function,

$$\langle 0|\, T^{*}\varPsi(y)\overline{\varPsi}(z)\,|0\rangle$$

$=$



$$= \int\frac{d^4k}{(2\pi)^4}\mathrm{e}^{-ik(y-z)}\Bigg[\frac{i}{\not{q}-m} + \int\frac{d^4l}{(2\pi)^4}\frac{i}{\not{q}-m}ie\gamma_\rho\frac{i}{\not{q}-\not{l}-m}ie\gamma_\sigma\frac{i}{\not{q}-m} \times\frac{-i}{l^2}\Big\{g^{\rho\sigma}+il^\rho\phi^\sigma(l)+i\phi^\rho(l)l^\sigma-l^\rho l^\sigma\phi(l)\phi(l)\Big\}+O(e^4)\Bigg]\ . \tag{3.27}$$

Note that the photon propagator has been replaced such that

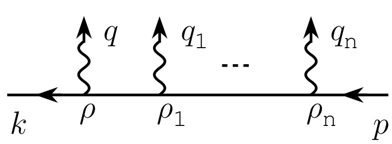
$$\frac{-i}{l^2}d^{\rho\sigma}(l)\longrightarrow\frac{-i}{l^2}\Big\{g^{\rho\sigma}+il^\rho\phi^\sigma(l)+i\phi^\rho(l)l^\sigma-l^\rho l^\sigma\phi(l)\phi(l)\Big\}\ . \tag{3.28}$$

This is our statement: the α-dependent propagator turns into a ϕ-dependent one.

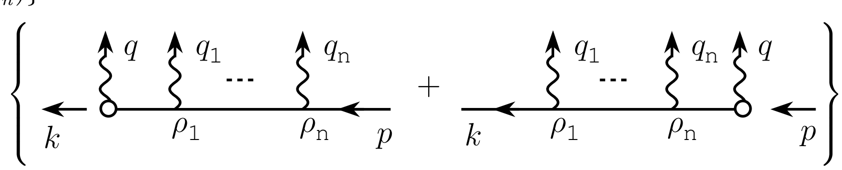
Similarly in a multi-point function (3.26), take a single fermion line to which n+1 vertices are attaching. With regard to a special vertex ρ out of which the momentum q flows, there arise two new contributions from $\phi^\rho(q)$. In terms of the notations in the previous section 2.4 it gives

$$\begin{aligned}
&i(-e)^{n+1}I^{\rho\rho_1\cdots\rho_n}(q,q_1,\cdots,q_n;k,p)\\
&+ie\phi^\rho(q)i(-e)^nI^{\rho_1\cdots\rho_n}(q_1,\cdots,q_n;k+q,p)\\
&+i(-e)^nI^{\rho_1\cdots\rho_n}(q_1,\cdots,q_n;k,p-q)ie\phi^\rho(q)\\
&=i(-e)^{n+1}\Big(\delta^\rho_\lambda+i\phi^\rho(q)q_\lambda\Big)I^{\lambda\rho_1\cdots\rho_n}(q,q_1,\cdots,q_n;k,p)
\end{aligned} \tag{3.29}$$

$$= \sum_{\substack{permutation\\ \{\binom{q}{\rho},\binom{q_1}{\rho_1},\cdots,\binom{q_n}{\rho_n}\}}}$$



$$+ \sum_{\substack{permutation\\ \{\binom{q_1}{\rho_1},\cdots,\binom{q_n}{\rho_n}\}}}$$



,

where use has been made of WT about I's (2.59) to the second expression. By applying the same manipulation to each vertex, the $n+1$-th photon amputated part of (3.26) becomes to

$$(3.26)\longrightarrow i(-e)^{n+1}\Big(\delta^\rho_\lambda+i\phi^\rho(q)q_\lambda\Big)\Big(\delta^{\rho_1}_{\lambda_1}+i\phi^{\rho_1}(q_1)q_{1_{\lambda_1}}\Big)\cdots\Big(\delta^{\rho_n}_{\lambda_n}+i\phi^{\rho_n}(q_n)q_{n_{\lambda_n}}\Big) \times I^{\lambda\lambda_1\cdots\lambda_n}(q,q_1,\cdots,q_n;k,p)\ . \tag{3.30}$$

Accordingly we find that each photon index, $I^{\rho\cdots}$, is modified to $\left(\delta^\rho_\kappa + i\phi^\rho(q)q_\kappa\right)I^{\kappa\cdots}$ as the result of adopting the gauge invariant electron Ψ, which, combined with the photon part (3.18), leads us to the result that the photon propagator is modified to

$$\begin{aligned} D^\phi_{\mu\nu}(q) &\equiv \left(\delta^\rho_\mu + i\phi^\rho(q)q_\mu\right)\frac{-i}{q^2}d_{\rho\sigma}(q)\left(\delta^\sigma_\nu + i\phi^\sigma(q)q_\nu\right) \\ &= \frac{-i}{q^2}\Big\{g_{\mu\nu} + iq_\mu\phi_\nu(q) + i\phi_\mu(q)q_\nu - q_\mu q_\nu\phi(q)\cdot\phi(q)\Big\} \ , \end{aligned} \tag{3.31}$$

where use has been made of the projection property (3.22). As expected, *gauge (α) dependence has been taken over by ϕ^μ.*

The covariant Landau gauge is realized by choosing $\phi^\mu(l)$ as

$$\phi^\mu(l) = \frac{il^\mu}{l^2} \ , \tag{3.32}$$

giving

$$D^L_{\mu\nu}(q) \equiv \frac{-i}{q^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \ . \tag{3.33}$$

Also the Coulomb gauge propagator is, in view of (3.24), given, by choosing

$$\phi^\mu(l) = \left(0, \frac{-i\boldsymbol{l}}{\boldsymbol{l}^2}\right) \ , \tag{3.34}$$

as

$$D^C_{\mu\nu}(q) \equiv \frac{-i}{l^2}\left(g_{\mu\nu} + \frac{\sum_j\left(g_{\nu j}l_\mu l^j + g_{\mu j}l^j l_\nu\right) - l_\mu l_\nu}{\boldsymbol{l}^2}\right) \ . \tag{3.35}$$

In this way, we have recognized that *building up gauge (BRS) invariant electron and photon is merely synonymous to fixing the gauge.*

Although all the operators in the expectation value,

$$\langle 0|\, T\Theta^{\mu\nu}(x)A^{\phi\lambda_1}(x_1)\cdots A^{\phi\lambda_n}(x_n)\Psi(y_1)\cdots\Psi(y_m)\overline{\Psi}(z_1)\cdots\overline{\Psi}(z_m)\,|0\rangle \ , \tag{3.36}$$

with $\Theta^{\mu\nu}(x)$ being given in (2.23), are BRS invariant, the value itself depends thus on ϕ. Even in this invariant approach there still need the physical state conditions (2.28) and (2.29) for proving ϕ-independence of $\Theta^{\mu\nu}(x)$.

In spite of the fact that the BRS invariant electron and photon are not so useful for the proof of gauge independence, they serve us as a probe into the structure of the theory. For example, the LSZ-mapping is easily clarified with the aid of Ψ:

$$\Psi(x) \mapsto Z_2^{1/2}\psi^{as}(x) + \text{higher orders} \ , \tag{3.37}$$

together with the photon sector

$$\begin{aligned} A_\mu(x) &\mapsto Z_3^{1/2}A^{as}_\mu(x) + \text{higher orders} \ , \\ B(x) &\mapsto Z_3^{-1/2}B^{as}(x) \ . \end{aligned} \tag{3.38}$$

Note that the relation (3.37) could be established with the aid of the invariant approach: since from (2.11) the asymptotic electron has been confirmed as BRS invariant.

As was stressed before, if the support of ϕ^μ is space-like, like in the case of the Dirac's electron, (3.37) can hold as the (weak) operator relation. Therefore strictly speaking, we can declare that the LSZ-mapping can be confirmed only in the case of Dirac's electron. This fact also tells us that in QED electron can behave as observable, whose statement could then be generalized to QCD as a trial for illustrating the dynamical mechanism of quark confinement [7].

4 Physical States in Functional Representation

In this section we discuss other physical states in terms of the functional representation. Also by use of that we build up the path integral formula in the Coulomb gauge and make an explicit transformation to the covariant gauge. We clarify the reason for this ability.

4.1 Other Physical States

Consider $A_0 = 0$ gauge in the conventional treatment [20]: all three components $\boldsymbol{A}$ are assumed dynamical and obey the commutation relations,

$$[\hat{A}_j(\mathbf{x}), \hat{E}_k(\mathbf{y})] = i\delta_{jk}\delta(\mathbf{x}-\mathbf{y}), \quad [\hat{A}_j(\mathbf{x}), \hat{A}_k(\mathbf{x})] = 0 = [\hat{E}_j(\mathbf{x}), \hat{E}_k(\mathbf{x})]; \quad (j,k=1,2,3)\ . \tag{4.1}$$

Here and hereafter the caret designates operators. The physical state condition (1.18) is then

$$\hat{\Phi}(\boldsymbol{x})|phys\rangle \equiv \left[\sum_{k=1}^{3}(\partial_k \hat{\boldsymbol{E}}_k(\boldsymbol{x})) + J_0(\boldsymbol{x})\right]|phys\rangle = 0\ , \tag{4.2}$$

where $J_\mu(x)$ is supposed as a c-number current. First this should be read such that *there is no gauge transformation in the physical space.* As was mentioned in the introduction, the representation of the physical state cannot be obtained within the usual Fock space since $\hat{\Phi}(\boldsymbol{x})$ is a local operator to result in $\hat{\Phi}(\boldsymbol{x}) = 0$ [18], but can be in the functional (Schrödinger) representation [21]:

$$\begin{gathered}\widehat{\boldsymbol{A}}(\boldsymbol{x})|\{\boldsymbol{A}\}\rangle = \boldsymbol{A}(\boldsymbol{x})|\{\boldsymbol{A}\}\rangle\ , \quad \widehat{\boldsymbol{E}}(\boldsymbol{x})|\{\boldsymbol{E}\}\rangle = \boldsymbol{E}(\boldsymbol{x})|\{\boldsymbol{E}\}\rangle\ , \\ \langle\{\boldsymbol{A}\}|\widehat{\boldsymbol{E}}(\boldsymbol{x}) = -i\frac{\delta}{\delta\boldsymbol{A}(\boldsymbol{x})}\langle\{\boldsymbol{A}\}|\ , \ldots\end{gathered} \tag{4.3}$$

To see the reason take the states, $|\{\boldsymbol{A}\}\rangle, |\{\boldsymbol{E}\}\rangle$, which can be constructed in terms of the Fock states. The creation and annihilation operators are given by

$$\begin{gathered}\widehat{\boldsymbol{A}}(\boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2|\boldsymbol{k}|}}(\mathbf{a}(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \mathbf{a}^\dagger(\boldsymbol{k})e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}) \\ [a_i(\boldsymbol{k}), a_j^\dagger(\boldsymbol{k}')] = \delta_{ij}\delta(\boldsymbol{k}-\boldsymbol{k}'), \quad [a_i(\boldsymbol{k}), a_j(\boldsymbol{k}')] = 0\ ,\end{gathered} \tag{4.4}$$

with the vacuum $|0\rangle$;

$$\mathbf{a}(\mathbf{k})|0\rangle = 0\ . \tag{4.5}$$

Now recall the quantum mechanical case [22]:

$$\begin{gathered}\hat{q}|q\rangle = q|q\rangle\ , \qquad \hat{p}|p\rangle = p|p\rangle\ , \\ \hat{q} = \frac{1}{\sqrt{2}}\left(a + a^\dagger\right)\ , \quad \hat{p} = \frac{1}{\sqrt{2}i}\left(a - a^\dagger\right)\ ; \qquad a|0\rangle = 0\ ,\end{gathered} \tag{4.6}$$

then

$$\begin{aligned}|q\rangle &= \frac{1}{\pi^{1/4}}\exp\left(-\frac{q^2}{2} + \sqrt{2}qa^\dagger - \frac{(a^\dagger)^2}{2}\right)|0\rangle\ , \\ |p\rangle &= \frac{1}{\pi^{1/4}}\exp\left(-\frac{p^2}{2} + \sqrt{2}ipa^\dagger + \frac{(a^\dagger)^2}{2}\right)|0\rangle\ .\end{aligned} \tag{4.7}$$

These bring us to

$$|\{\boldsymbol{A}\}\rangle \simeq \exp\Big[-\frac{1}{2}\int d^3\boldsymbol{x}\; d^3\boldsymbol{y}\; \boldsymbol{A}(\boldsymbol{x})K(\boldsymbol{x}-\boldsymbol{y})\boldsymbol{A}(\boldsymbol{y}) \\ +\int d^3\boldsymbol{x}\int d^3\boldsymbol{k}\;\sqrt{\frac{2|\boldsymbol{k}|}{(2\pi)^3}}\;\boldsymbol{A}(\boldsymbol{x})\cdot\mathbf{a}^\dagger(\boldsymbol{k})e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} - \frac{1}{2}\int d^3\boldsymbol{k}\;\mathbf{a}^\dagger(\boldsymbol{k})\cdot\mathbf{a}^\dagger(-\boldsymbol{k})\Big]|0\rangle \;, \tag{4.8}$$

where

$$K(\boldsymbol{x}) \equiv \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}|\boldsymbol{k}|e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \;, \tag{4.9}$$

which is apparently divergent so we must introduce some cut-off. The physical state in the functional representation is thus found as

$$\langle\{\boldsymbol{A}\}|\hat{\Phi}(\boldsymbol{x})|phys\rangle = \left(-i\boldsymbol{\nabla}\frac{\delta}{\delta\boldsymbol{A}(\boldsymbol{x})} - J_0(\boldsymbol{x})\right)\Psi_{\rm phys}[A] = 0 \;, \tag{4.10}$$

where

$$\Psi_{\rm phys}[A] \equiv \langle\{\boldsymbol{A}\}|phys\rangle \;. \tag{4.11}$$

Now we can see the reason for having the physical state in this case. Within a single Fock state the physical state condition (4.2) merely implies $\hat{\Phi}(\boldsymbol{x}) = 0$ but *the functional representation consists of infinitely many collections of inequivalent Fock spaces*: since the inner product of $|\{\boldsymbol{A}\}\rangle$ (4.8), to the Fock vacuum is found to be

$$\langle\{\boldsymbol{A}\}|0\rangle \sim \exp\left[-\frac{1}{2}\int d^3\boldsymbol{x}\; d^3\boldsymbol{y}\; \boldsymbol{A}(\boldsymbol{x})\; K(\boldsymbol{x}-\boldsymbol{y})\boldsymbol{A}(\boldsymbol{y})\right] \\ = \exp\left[-\frac{1}{2}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\;|\boldsymbol{k}|\;\boldsymbol{A}(\boldsymbol{k})\boldsymbol{A}(-\boldsymbol{k})\right] \longrightarrow 0\;;{}^\forall\{\boldsymbol{A}\}\;, \tag{4.12}$$

when the cut-off becomes infinity. This happens in *any value of* $\boldsymbol{A}(\boldsymbol{x})$. Therefore (apart from the mathematical rigorousness of that) any local first class constraint can be realized by means of the functional representation. Furthermore the fact that the functional representation contains an infinite set of the Fock states enables us to perform an explicit gauge transformation and prove gauge independence without recourse to any physical state conditions.

4.2 Proof of Gauge Independence by Path Integral

Recall that the path integral formula can be obtained with the aid of the functional representation. It then might be easily convinced that *we can move freely from one gauge to another in the path integral* [8, 23].

Take the Coulomb case. The Hamiltonian is given by

$$H = \int d^3\boldsymbol{x}\left[\frac{1}{2}{\boldsymbol{E}_{\rm T}}^2 + \frac{1}{2}(\boldsymbol{\nabla}_i\boldsymbol{A}_{\rm T})^2 + \frac{1}{2}J_0\frac{1}{\boldsymbol{\nabla}^2}J_0 - \boldsymbol{J}\cdot\boldsymbol{A}\right] \;, \tag{4.13}$$

where the third term in the right hand side is the nonlocal Coulomb energy term with J_μ being assumed as c-number sources. The equal time commutation relations are

$$[\hat{A}_i(x), \hat{E}_j(y)] = i\left(\delta_{ij} - \frac{\boldsymbol{\nabla}_i\boldsymbol{\nabla}_j}{\boldsymbol{\nabla}^2}\right)(\boldsymbol{x},\boldsymbol{y}) \equiv i\mathbf{P}_{ij}(\boldsymbol{x},\boldsymbol{y}) \;, \quad [\hat{A}_i, \hat{A}_j] = [\hat{E}_i, \hat{E}_j] = 0 \;, \tag{4.14}$$

where

$$\mathbf{P}_{ij}(\mathbf{x},\mathbf{y}) = \int d^3\boldsymbol{k}\ e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}\left(\delta_{ij} - \frac{k_i k_j}{\boldsymbol{k}^2}\right)\ , \tag{4.15}$$

which can be diagonalized [24] by means of $\boldsymbol{S}$ such that

$$\boldsymbol{S}\boldsymbol{P}\boldsymbol{S}^T = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix}, \tag{4.16}$$

where T designates the transpose. Explicitly

$$\boldsymbol{S}_{1i} = \boldsymbol{n}_i\ , \quad \boldsymbol{S}_{2i} = \left(\frac{\boldsymbol{\nabla}}{|\boldsymbol{\nabla}|}\times\boldsymbol{n}\right)_i\ , \quad \boldsymbol{S}_{3i} = \frac{\boldsymbol{\nabla}_i}{|\boldsymbol{\nabla}|}\ , \tag{4.17}$$

where $\boldsymbol{n}$ is some vector perpendicular to $\boldsymbol{\nabla}$; $\boldsymbol{\nabla}\cdot\boldsymbol{n} = 0$. Owing to $\boldsymbol{S}$ two components can be picked out so that we can write (omitting the caret)

$$(\boldsymbol{S}\boldsymbol{A})_a = \widetilde{A}_a\ , \quad (\boldsymbol{S}\boldsymbol{E})_a = \widetilde{E}_a\ , \qquad (a = 1,2)\ , \tag{4.18}$$

and

$$[\widetilde{A}_a, \widetilde{E}_b] = i\delta_{ab}\ , \quad (a,b = 1,2)\ . \tag{4.19}$$

Also by noting that

$$\boldsymbol{E}_{\mathrm{T}}^2 = \boldsymbol{E}_i\boldsymbol{P}_{ij}\boldsymbol{E}_j = (\widetilde{E}_a)^2\ , \tag{4.20}$$

we obtain

$$H(t) = \int d^3\boldsymbol{x}\left[\frac{1}{2}(\widetilde{E}_a)^2 + \frac{1}{2}(\boldsymbol{\nabla}_i\widetilde{A}_a)^2 + \frac{1}{2}J_0\frac{1}{\boldsymbol{\nabla}^2}J_0 - \widetilde{J}_a\widetilde{A}_a\right]\ , \tag{4.21}$$

where $\widetilde{J}_a \equiv (\boldsymbol{S}\boldsymbol{J})_a$ and to specify the explicit time dependence through the Coulomb energy term we have written the Hamiltonian as $H(t)$. The summation convention for the repeated indices must be implied.

The starting point of the path integral is [23],

$$Z(T) \equiv \lim_{N\to\infty}(\mathbf{I} - i\Delta t H_N)(\mathbf{I} - i\Delta t H_{N-1})\cdots(\mathbf{I} - i\Delta t H_1)\ , \tag{4.22}$$

where $\Delta t \equiv T/N$ and $H_j \equiv H(j\Delta t)$. (Usually the Euclidean technique, $T \to -iT$, must be used [23]. Here in order to illustrate the way to get the path integral expression we keep i in the trace formula. Also to make a whole discussion well-defined it is necessary to discretize space $\boldsymbol{x}$ but in the following the continuum expression is employed only for the notational simplicity.) The essential ingredients[1] are the functional (Schrödinger) representation (4.3) together with

$$\begin{aligned} &\int \mathcal{D}\widetilde{A}_a(\boldsymbol{x})|\{\widetilde{A}\}\rangle\langle\{\widetilde{A}\}| = \mathbf{I}\ , \\ &\int \mathcal{D}\widetilde{E}_a(\boldsymbol{x})|\{\widetilde{E}\}\rangle\langle\{\widetilde{E}\}| = \mathbf{I}\ ; \end{aligned} \tag{4.23}$$

[1] Path integral expressions for relativistic field for instance, $\phi(x)$, obtained via holomorphic (canonical coherent) representation [25] in terms of the creation and the annihilation operator $a(\boldsymbol{k})^\dagger, a(\boldsymbol{k})$, must suffer from nonlocality whenever going back to the $\phi(x)$-representation, due to the mixing of particle and anti-particle. In order to get rid of this difficulty, we should start with the field $\phi(x)$-diagonal representation which is nothing but the functional Schrödinger one.

$$\langle\{\widetilde{E}\}|\{\widetilde{A}\}\rangle = \left(\prod_{\boldsymbol{x}} \frac{1}{2\pi}\right) \exp\left[-i\int d^3\boldsymbol{x}\widetilde{E}_a(\boldsymbol{x})\widetilde{A}_a(\boldsymbol{x})\right] \ , \tag{4.24}$$

whose (infinite) constant will be absorbed in the functional measure in the following. Inserting (4.23) into (4.22) successively and using (4.24), we obtain the path integral expression,

$$\begin{aligned}
Z(T) &= \lim_{N\to\infty}\prod_{j=1}^{N}\int \mathcal{D}\widetilde{A}_a(\boldsymbol{x};j)\mathcal{D}\widetilde{E}_a(\boldsymbol{x};j)\exp\Bigg[i\Delta t\int d^3\boldsymbol{x} \\
&\quad\times\Bigg\{\widetilde{E}_a(\boldsymbol{x};j)\frac{\left(\widetilde{A}_a(\boldsymbol{x};j)-\widetilde{A}_a(\boldsymbol{x};j-1)\right)}{\Delta t} - \frac{1}{2}\left(\widetilde{E}_a(\boldsymbol{x};j)\right)^2 + \frac{1}{2}\left(\boldsymbol{\nabla}_i\widetilde{A}_a(\boldsymbol{x};j)\right)^2 \\
&\quad+\frac{1}{2}\int d^3\boldsymbol{y}\ J_0(\boldsymbol{x};j)\frac{1}{\boldsymbol{\nabla}^2}(\boldsymbol{x},\boldsymbol{y})J_0(\boldsymbol{y};j) - \widetilde{J}_a(\boldsymbol{x};j)\widetilde{A}_a(\boldsymbol{x};j)\Bigg\}\Bigg] \qquad (4.25)\\
&= \int \mathcal{D}\widetilde{A}_a(x)\mathcal{D}\widetilde{E}_a(x)\exp\Bigg[i\int d^4x\Bigg\{\widetilde{E}_a\dot{\widetilde{A}}_a \\
&\qquad -\left(\frac{1}{2}\left(\widetilde{E}_a\right)^2 + \frac{1}{2}\left(\boldsymbol{\nabla}_i\widetilde{A}_a\right)^2 + \frac{1}{2}J_0\frac{1}{\boldsymbol{\nabla}^2}J_0 - \widetilde{J}_a\widetilde{A}_a\right)\Bigg\}\Bigg] \ ,
\end{aligned}$$

where we have again employed a continuous expression in the final line. (The periodic boundary condition $\widetilde{A}_a(\boldsymbol{x};T) = \widetilde{A}_a(\boldsymbol{x};0)$ is now irrelevant so we have not specified it.) Now inserting

$$1 = \int \mathcal{D}\widetilde{A}_3\mathcal{D}\widetilde{E}_3\delta(\widetilde{A}_3)\delta(\widetilde{E}_3) \tag{4.26}$$

into (4.25) and changing the variables to the original $\boldsymbol{A}$ and $\boldsymbol{E}$ in view of (4.17) and (4.18), we obtain

$$\begin{aligned}
Z(T) = \int \mathcal{D}\boldsymbol{E}\ \mathcal{D}\boldsymbol{A}\ (\det\boldsymbol{\nabla}^2)\delta(\boldsymbol{\nabla}\cdot\boldsymbol{E})\delta(\boldsymbol{\nabla}\cdot\boldsymbol{A})\exp\Bigg[i\int d^4x\Big\{\boldsymbol{E}\cdot\dot{\boldsymbol{A}} \\
-(\frac{1}{2}\boldsymbol{E}^2 + \frac{1}{4}(F_{ij})^2 + \frac{1}{2}J_0\frac{1}{\boldsymbol{\nabla}^2}J_0 - \boldsymbol{J}\cdot\boldsymbol{A})\Big\}\Bigg] \ .
\end{aligned} \tag{4.27}$$

With the use of Fourier transformation of the delta function,

$$\delta(\boldsymbol{\nabla}\cdot\boldsymbol{E}) = \int \mathcal{D}\beta\exp\left[i\int d^4x\ \beta\ \boldsymbol{\nabla}\cdot\boldsymbol{E}\right] \ , \tag{4.28}$$

the integration with respect to $\boldsymbol{E}$ can be performed to obtain

$$\begin{aligned}
Z(T) = \int \mathcal{D}\boldsymbol{A}\ \mathcal{D}\beta\ (\det\boldsymbol{\nabla}^2)\delta(\boldsymbol{\nabla}\cdot\boldsymbol{A}) \\
\times\exp\left[i\int d^4x\Big\{\frac{1}{2}(\dot{\boldsymbol{A}}-\boldsymbol{\nabla}\beta)^2 - \frac{1}{4}(F_{ij})^2 - \frac{1}{2}J_0\frac{1}{\boldsymbol{\nabla}^2}J_0 + \boldsymbol{J}\cdot\boldsymbol{A}\Big\}\right] \ .
\end{aligned} \tag{4.29}$$

Here by reviving A_0 in the form of

$$A_0 = \beta + \frac{J_0}{\boldsymbol{\nabla}^2} \tag{4.30}$$

the nonlocal (as well as instantaneous) Coulomb interaction is eliminated to leave the final form;

$$Z(T) = \int \mathcal{D}A_\mu\ (\det\boldsymbol{\nabla}^2)\ \delta(\boldsymbol{\nabla}\cdot\boldsymbol{A})\exp\left[-i\int d^4x\Big\{\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu\Big\}\right] \ . \tag{4.31}$$

Here using the relation,

$$\delta(\boldsymbol{\nabla}\cdot\boldsymbol{A}) \sim \lim_{\alpha\to 0} \exp\left[-\frac{i}{2\alpha}\int d^4x\ (\boldsymbol{\nabla}\cdot\boldsymbol{A})^2\right], \tag{4.32}$$

to (4.31) then integrating with respect to the gauge fields we obtain

$$Z(T) = \exp\left[i\int d^4x\ d^4y\ \frac{1}{2}J^\mu(x)D_{\mu\nu}(x-y)J^\nu(y)\right], \tag{4.33}$$

where $D_{\mu\nu}(x)$ is the Fourier transformation of the propagator (3.35).

It is now a simple task to go to another gauge [23]. Suppose the new gauge condition is given by

$$\partial^\mu A'_\mu(x) = f(x)\ , \tag{4.34}$$

where $f(x)$ is an arbitrary function. The gauge transformation is

$$A'_\mu(x) = A_\mu(x) + \partial_\mu\chi(x)\ . \tag{4.35}$$

In order to find such $\chi(x)$, first we rewrite (4.34) as

$$\dot{A}'_0(x) = f(x) - \boldsymbol{\nabla}\cdot\boldsymbol{A}'(x)\ , \tag{4.36}$$

and substituting (4.35) into this to find

$$\Box\chi(x) = f(x) - \dot{A}_0(x)\ . \tag{4.37}$$

Thus the Jacobian is read as

$$\det\left(\frac{\delta A'_\mu}{\delta A_\nu}\right) = \det\left(\delta^\nu_\mu - \delta^\nu_0\frac{\partial_\mu\partial_0}{\Box}\right) = \det(-\boldsymbol{\nabla}^2\Box^{-1})\ , \tag{4.38}$$

giving

$$\mathcal{D}A_\mu(\det\boldsymbol{\nabla}^2)\delta(\boldsymbol{\nabla}\cdot\boldsymbol{A}) = \mathcal{D}A'_\mu(\det\Box)\delta(\partial^\mu A'_\mu - f)\ , \tag{4.39}$$

where the minus sign in the determinant is irrelevant so we have dropped it. Therefore

$$Z(T) = \int\mathcal{D}A_\mu(\det\Box)\delta(\partial^\mu A_\mu - f)\exp\left[-i\int d^4x\left\{\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu\right\}\right]\ . \tag{4.40}$$

In this way, the gauge transformation in the path integral expression can be performed straightforwardly according to the fact that *the functional representation has infinitely many inequivalent representations.*

5 Discussion

In this paper, we recognize that the Belinfante's energy-momentum tensor is gauge independent for all orders under the LSZ asymptotic conditions in §2. Meanwhile, we know that to pick up the gauge invariant electron or photon is merely synonymous to fix the gauge: the transverse part, $\boldsymbol{A}_{\rm T}, \boldsymbol{\nabla}\cdot\boldsymbol{A}_{\rm T} = 0$, is gauge invariant; which is equivalent to the Coulomb gauge. In this case, ψ itself becomes gauge invariant. It has been sometimes argued that gauge symmetry is not a symmetry rather a redundancy [26]. There need only two components but in order to recover the rotational as well as the Lorentz invariance, spurious two components have been added. These

spurious components move around under gauge transformations leaving the physical component unchanged. Therefore, our observation in §3 is natural; that is, picking up the gauge invariant quantities leaves the Lorentz or rotational non-invariance. (If the Lorentz invariance is kept respectable, the negative metric must be introduced, so that operators themselves lose their significance without recourse to physical state conditions[27].) The situation reminds us of that of lattice gauge theory [28]; in which the gauge invariance has been maintained at the sacrifice of the Lorentz (Euclidean) as well as the rotational invariance. The method provides us the non-perturbative treatment in a gauge invariant way, leading to confinement in terms of an area law [28] by the help of an analogy with the critical phenomena in statistical mechanics. However, more physical and concrete views must be necessary in order to understand the confinement problem thoroughly: for example, the existence of Dirac's physical electron assures us that there is no confinement in QED. Therefore if proof could be given in QCD that the physical quark fields cannot be built up, the issue is resolved. The Gribov ambiguity [19] would be the cornerstone of the proof: canonical commutation relations (as a result of gauge fixing) between gauge fields can only hold within some small region around a point, where the coupling constant is small enough. Enlarging the region, we see that there takes part another gauge configuration, bringing us to the impossibility of observing quarks [7].

According to the discussion in §4 the path integral formulation would be most suitable for treating gauge transformation; since infinitely many Hilbert spaces compose the functional representation which is the basic of the path integral formula. The issue is then how to patch those small regions together to cover the whole functional space. There might be some hints from the recent observations in quantum mechanics on nontrivial manifolds [29] and the path integral formula for a generic constraint [30].

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A The Covariant LSZ-Formalism in QED

In this appendix we review the asymptotic behavior of photon fields and the LSZ reduction formula given by Nakanishi [13] for a self-contained purpose.

A.1 Asymptotic Photon Field

In order to know the behavior of asymptotic fields, it is necessary to investigate the Heisenberg fields. Then start with the Nakanishi-Lautrup Lagrangian (1.7) in a renormalized form,

$$\mathcal{L} = -\frac{1}{4}Z_3 F^{\mu\nu}F_{\mu\nu} + \overline{\psi}(\frac{i}{2}\overleftrightarrow{\not\partial} + eZ_3^{\frac{1}{2}}\not A - m)\psi - A^\mu\,\partial_\mu B + \frac{\alpha}{2}B^2\ , \tag{A.1}$$

where all quantities have been assumed renormalized except ψ, m, and e, and Z_3 is the wave function renormalization constant for photon. The equations of motion are

$$
\begin{aligned}
&\Box A_\mu - (1-\alpha)\,\partial_\mu B = \boldsymbol{j}_\mu \ , \\
&\partial^\mu A_\mu + \alpha B = 0 \ , \\
&\Box B = 0 \ ,
\end{aligned}
\tag{A.2}
$$

where

$$
\boldsymbol{j}_\mu = {Z_3}^{-\frac{1}{2}} j_\mu - \left(1 - {Z_3}^{-1}\right) \partial_\mu B \ , \tag{A.3}
$$

and

$$
j_\mu \equiv -e\overline{\psi}\gamma_\mu\psi \ . \tag{A.4}
$$

The four-dimensional commutation relations among A_μ's and B are found as

$$
\begin{aligned}
&[\, A_\mu(x)\,, B(y)\,] = -i\,\partial_\mu^x D(x-y) \ , \\
&[\, B(x)\,, B(y)\,] = 0 \ , \\
&[\, B(x)\,, j_\mu(y)\,] = 0 \ ,
\end{aligned}
\tag{A.5}
$$

where $D(x)$ is the invariant delta function,

$$
D(x) \equiv \int \frac{d^4p}{(2\pi)^3 i} \epsilon(p_0)\delta(p^2)\,\mathrm{e}^{-ipx} \ . \tag{A.6}
$$

In order to obtain those for A_μ's compute first $\langle 0|\, j_\mu(x) j_\nu(y)\,|0\rangle$ and then utilize (A.2), (A.5), as well as

$$
[\, A_k(x_0\,, \boldsymbol{x})\,, \dot{A}_l(x_0\,, \boldsymbol{y})\,] = -\frac{i}{Z_3} g_{kl}\delta^3(\boldsymbol{x}-\boldsymbol{y}) \ , \tag{A.7}
$$

to find [31]

$$
\begin{aligned}
&\langle 0|\,[\, A_\mu(x)\,, A_\nu(y)\,]\,|0\rangle \\
&= -i\left(g_{\mu\nu} - K\,\partial_\mu\partial_\nu\right) D(x-y) + i\,(1-\alpha)\,\partial_\mu\partial_\nu E(x-y) \\
&\quad - \frac{i}{Z_3}\int_{+0}^{\infty} ds\,\sigma(s)\left(g_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{s}\right)\Delta(x-y;s) \ ,
\end{aligned}
\tag{A.8}
$$

where

$$
K \equiv \frac{1}{Z_3}\int_{+0}^{\infty} ds \frac{\sigma(s)}{s} \ , \tag{A.9}
$$

$$
Z_3 \equiv 1 - \int_{+0}^{\infty} ds\sigma(s) \ , \tag{A.10}
$$

with $\sigma(s)$ being the spectral function, and $\Delta(x;s)$ and $E(x)$ are expressed as

$$
\begin{aligned}
&\Delta(x;s) \equiv \int \frac{d^4p}{(2\pi)^3 i}\epsilon(p_0)\delta(p^2-s)\,\mathrm{e}^{-ipx} \ , \\
&E(x) \equiv \int \frac{d^4p}{(2\pi)^3 i}\epsilon(p_0)\delta'(p^2)\,\mathrm{e}^{-ipx} \ ; \quad \delta'(a) \equiv \frac{d}{da}\delta(a) \ .
\end{aligned}
\tag{A.11}
$$

Once the four-dimensional commutation relations is obtained, so can be those for the asymptotic fields, $A_\mu^{as}(x)$, $B^{as}(x)$, by simply throwing away the continuous spectrum part in (A.5) and (A.8):

$$
\begin{aligned}
&[\, A_\mu^{as}(x)\,, A_\nu^{as}(y)\,] = -i\left(g_{\mu\nu} - K\,\partial_\mu\partial_\nu\right) D(x-y) + i\,(1-\alpha)\,\partial_\mu\partial_\nu E(x-y)\ ,\\
&[\, A_\mu^{as}(x)\,, B^{as}(y)\,] = -i\,\partial_\mu^x D(x-y)\ ,\\
&[\, B^{as}(x)\,, B^{as}(y)\,] = 0\ .
\end{aligned}
\tag{A.12}
$$

From this the equations of motion for the asymptotic fields reads

$$
\begin{aligned}
\Box A_\mu^{as} - (1-\alpha)\,\partial_\mu B^{as} &= 0\ ,\\
\partial^\mu A_\mu^{as} + \alpha B^{as} &= 0\ ,\\
\Box B^{as} &= 0\ .
\end{aligned}
\tag{A.13}
$$

It should be noted that the canonical structure (A.12) differs from that of the free theory in which the equations of motion is the same as (A.13) but the commutation relations is given without K! The Lagrangian leading to (A.12) as well as (A.13) is found as

$$
\mathcal{L}^{as} = -\frac{1}{4}F^{as\mu\nu}F^{as}{}_{\mu\nu} - A^{as\mu}\,\partial_\mu B^{as} + \frac{K}{2}\partial_\mu B^{as}\partial^\mu B^{as} + \frac{\alpha}{2}\left(B^{as}\right)^2\ . \tag{A.14}
$$

Note the existence of the kinetic term of B.

A.2 Wave Functions

To obtain the LSZ formula, there needs to construct wave functions. When $\alpha \neq 1$ three sets of positive frequency functions $\{h_{\boldsymbol{k}\sigma}{}^\mu(x)\}$, $\{f_{\boldsymbol{k}\sigma}{}^\mu(x)\}$ and $\{g_{\boldsymbol{k}}(x)\}$ must be prepared to meet the equations,

$$
\begin{aligned}
&\Box^2 A_\mu(x) = 0\ ,\\
&\Box B(x) = 0\ .
\end{aligned}
\tag{A.15}
$$

Those then must obey

$$
\begin{aligned}
&\Box h_{\boldsymbol{k}\sigma}{}^\mu(x) = f_{\boldsymbol{k}\sigma}{}^\mu(x)\ ,\\
&\Box f_{\boldsymbol{k}\sigma}{}^\mu(x) = 0\ ,\\
&\Box g_{\boldsymbol{k}}(x) = 0\ ,
\end{aligned}
\tag{A.16}
$$

where $\boldsymbol{k}$ and σ denote the momentum and the polarization of photon respectively. An explicit representation is obtained by an orthonormal set, $\{\varphi_{\boldsymbol{k}}(\boldsymbol{p})\}$,

$$
\sum_{\boldsymbol{k}} \varphi_{\boldsymbol{k}}(\boldsymbol{p})\,\varphi_{\boldsymbol{k}}{}^*(\boldsymbol{q}) = \delta(\boldsymbol{p}-\boldsymbol{q})\ , \qquad \int d^3p\,\varphi_{\boldsymbol{k}}{}^*(\boldsymbol{p})\,\varphi_{\boldsymbol{l}}(\boldsymbol{p}) = \delta_{\boldsymbol{kl}}\ , \tag{A.17}
$$

and by a polarization vector $\xi_\sigma{}^\mu(\boldsymbol{p})$,

$$
\sum_{\sigma=0}^{3}\sum_{\tau=0}^{3} \xi_\sigma{}^\mu(\boldsymbol{p})\eta_{\sigma\tau}\xi_\tau{}^\nu(\boldsymbol{p}) = g^{\mu\nu}\ , \qquad \sum_{\mu=0}^{3} \xi_\sigma{}^\mu(\boldsymbol{p})\,\xi_{\tau\mu}(\boldsymbol{p}) = \eta_{\sigma\tau}\ , \tag{A.18}
$$

where $\mathrm{diag}(\eta_{\sigma\tau}) = (1,-1,-1,-1)$; $\mathrm{diag}(g_{\mu\nu}) = (1,-1,-1,-1)$. With these we have

$$
\begin{aligned}
g_{\boldsymbol{k}}(x) &= \int \frac{d^3p}{\sqrt{(2\pi)^3 2p_0}}\, \varphi_{\boldsymbol{k}}(\boldsymbol{p})\, \mathrm{e}^{-ipx} \ , \qquad p_0 = |\boldsymbol{p}| \ , \\
f_{\boldsymbol{k}\sigma}{}^{\mu}(x) &= \int \frac{d^3p}{\sqrt{(2\pi)^3 2p_0}}\, \xi_\sigma{}^{\mu}(\boldsymbol{p})\, \varphi_{\boldsymbol{k}}(\boldsymbol{p})\, \mathrm{e}^{-ipx} \ , \qquad p_0 = |\boldsymbol{p}| \ , \\
h_{\boldsymbol{k}\sigma}{}^{\mu}(x) &= \frac{1}{2}\left(\boldsymbol{\nabla}^2\right)^{-1}\left\{\left(x_0\,\partial_0 - \frac{3}{2}\right) f_{\boldsymbol{k}\sigma}{}^{\mu}(x) + g^{\mu 0} f_{\boldsymbol{k}\sigma}{}^{0}(x)\right\} \ .
\end{aligned}
\tag{A.19}
$$

Now it is almost straightforward to see that the following relations hold:

$$
\begin{aligned}
\sum_{\boldsymbol{k}} g_{\boldsymbol{k}}(x)\, g_{\boldsymbol{k}}{}^{*}(y) &= iD^{(+)}(x-y) \ , \\
\sum_{\boldsymbol{k}} \sum_{\sigma=0}^{3} \sum_{\tau=0}^{3} f_{\boldsymbol{k}\sigma}{}^{\mu}(x)\, \eta_{\sigma\tau}\, f_{\boldsymbol{k}\tau}{}^{\nu *}(y) &= i g^{\mu\nu} D^{(+)}(x-y) \ , \\
\sum_{\boldsymbol{k}} \sum_{\sigma=0}^{3} \sum_{\tau=0}^{3} \overline{h}_{\boldsymbol{k}\sigma}(x)\, \eta_{\sigma\tau}\, \overline{h}^{*}_{\boldsymbol{k}\tau}(y) &= -iE^{(+)}(x-y) \ ,
\end{aligned}
\tag{A.20}
$$

where $E^{(+)}(x-y)$ and $D^{(+)}(x-y)$ are the positive frequency parts of $E(x-y)$ and $D(x-y)$ respectively, and

$$
\overline{h}_{\boldsymbol{k}\sigma}(x) \equiv \partial_\mu h_{\boldsymbol{k}\sigma}{}^{\mu}(x) \qquad \text{also} \qquad \overline{f}_{\boldsymbol{k}\sigma}(x) \equiv \partial_\mu f_{\boldsymbol{k}\sigma}{}^{\mu}(x) \ . \tag{A.21}
$$

(There needs some regularization to define $E^{(+)}(x-y)$ because of its logarithmic divergence. However this divergence goes out when differentiated with respect to x_μ.)

There hold additional relations:

$$
\begin{aligned}
i\int d^3x f_{\boldsymbol{k}\sigma}{}^{\mu *}(x)\, \overleftrightarrow{\partial}_0 f_{\boldsymbol{l}\tau\mu}(x) &= \delta_{\boldsymbol{k}\boldsymbol{l}}\, \eta_{\sigma\tau} \ , \\
i\int d^3x g_{\boldsymbol{k}}{}^{*}(x)\, \overleftrightarrow{\partial}_0 g_{\boldsymbol{l}}(x) &= \delta_{\boldsymbol{k}\boldsymbol{l}} \ .
\end{aligned}
\tag{A.22}
$$

A.3 Fock State and the LSZ formula

Before constructing the LSZ formula, we define the creation and annihilation operators,

$$
\begin{aligned}
\mathcal{A}^{as}_{\boldsymbol{k}\sigma}{}^{\dagger} &\equiv -i\int d^3x \left\{ f_{\boldsymbol{k}\sigma}{}^{\mu}(x)\, \overleftrightarrow{\partial}_0 A^{as}_{\mu}(x) + h_{\boldsymbol{k}\sigma}{}^{\mu}(x)\, \overleftrightarrow{\partial}_0 \Box A^{as}_{\mu}(x) \right\} \ , \\
\mathcal{A}^{as}_{\boldsymbol{k}\sigma} &\equiv i\int d^3x \left\{ f_{\boldsymbol{k}\sigma}{}^{\mu *}(x)\, \overleftrightarrow{\partial}_0 A^{as}_{\mu}(x) + h_{\boldsymbol{k}\sigma}{}^{\mu *}(x)\, \overleftrightarrow{\partial}_0 \Box A^{as}_{\mu}(x) \right\} \ ,
\end{aligned}
\tag{A.23}
$$

$$
\begin{aligned}
\mathcal{B}^{as\dagger}_{\boldsymbol{k}} &\equiv -i\int d^3x g_{\boldsymbol{k}}(x)\, \overleftrightarrow{\partial}_0 B^{as}(x) \ , \\
\mathcal{B}^{as}_{\boldsymbol{k}} &\equiv i\int d^3x g_{\boldsymbol{k}}{}^{*}(x)\, \overleftrightarrow{\partial}_0 B^{as}(x) \ .
\end{aligned}
\tag{A.24}
$$

The asymptotic one-photon states are given by

$$
\begin{aligned}
|\boldsymbol{k}\sigma; as\rangle &\equiv \mathcal{A}^{as}_{\boldsymbol{k}\sigma}{}^{\dagger}\, |0\rangle \ ; \qquad \mathcal{A}^{as}_{\boldsymbol{k}\sigma}\, |0\rangle = 0 \ , \\
|\boldsymbol{k}; as\rangle &\equiv \mathcal{B}^{as\dagger}_{\boldsymbol{k}}\, |0\rangle \ ; \qquad \mathcal{B}^{as}_{\boldsymbol{k}}\, |0\rangle = 0 \ .
\end{aligned}
\tag{A.25}
$$

Those satisfy, by means of (A.12),

$$
\begin{aligned}
[\, \mathcal{A}^{as}_{\boldsymbol{k}\sigma} \,, \mathcal{A}^{as\dagger}_{\boldsymbol{l}\tau} \,] &= -\delta_{\boldsymbol{kl}}\, \eta_{\sigma\tau} - iK \int d^3x \overline{f}^{\,*}_{\boldsymbol{k}\sigma}(x) \overleftrightarrow{\partial_0} \overline{f}_{\boldsymbol{l}\tau}(x) \\
&\qquad -i(1-\alpha) \int d^3x \Big\{ \overline{f}^{\,*}_{\boldsymbol{k}\sigma}(x) \overleftrightarrow{\partial_0} \overline{h}_{\boldsymbol{l}\tau}(x) + \overline{h}^{\,*}_{\boldsymbol{k}\sigma}(x) \overleftrightarrow{\partial_0} \overline{f}_{\boldsymbol{l}\tau}(x) \Big\} \;, \\
[\, \mathcal{A}^{as}_{\boldsymbol{k}\sigma} \,, \mathcal{B}^{as\dagger}_{\boldsymbol{l}} \,] &= i \int d^3x \overline{f}^{\,*}_{\boldsymbol{k}\sigma}(x) \overleftrightarrow{\partial_0} g_{\boldsymbol{l}}(x) \;, \\
[\, \mathcal{B}^{as}_{\boldsymbol{k}} \,, \mathcal{A}^{as\dagger}_{\boldsymbol{l}\tau} \,] &= i \int d^3x g_{\boldsymbol{k}}{}^{*}(x) \overleftrightarrow{\partial_0} \overline{f}_{\boldsymbol{l}\tau}(x) \;, [\, \mathcal{B}^{as}_{\boldsymbol{k}} \,, \mathcal{B}^{as\dagger}_{\boldsymbol{l}} \,] = 0 \;.
\end{aligned}
\tag{A.26}
$$

Armed with these we can construct the LSZ formula: to begin with, we rewrite (A.23) as

$$
\begin{aligned}
\mathcal{A}^{as}_{\boldsymbol{k}\sigma}{}^{\dagger} &= -i \int d^3x \Big\{ f_{\boldsymbol{k}\sigma}{}^{\mu}(x) \overleftrightarrow{\partial_0} A^{as}_{\mu}(x) - (1-\alpha) [\overline{h}_{\boldsymbol{k}\sigma}(x) \overleftrightarrow{\partial_0} + f_{\boldsymbol{k}\sigma}{}^{0}(x)] B^{\,as}(x) \Big\} \;, \\
\mathcal{A}^{as}_{\boldsymbol{k}\sigma} &= i \int d^3x \Big\{ f_{\boldsymbol{k}\sigma}{}^{\mu *}(x) \overleftrightarrow{\partial_0} A^{as}_{\mu}(x) - (1-\alpha)\, [\overline{h}^{\,*}_{\boldsymbol{k}\sigma}(x) \overleftrightarrow{\partial_0} + f_{\boldsymbol{k}\sigma}{}^{0*}(x)] B^{\,as}(x) \Big\} \;,
\end{aligned}
\tag{A.27}
$$

where use has been made of the equations of motion (A.13). Under the asymptotic condition $A_\mu \overset{|t|\to\infty}{\longrightarrow} A^{as}_\mu$, we obtain

$$
\begin{aligned}
\mathcal{A}^{out\dagger}_{\boldsymbol{k}\sigma} T(\mathcal{O}) - T(\mathcal{O})\, \mathcal{A}^{in\;\dagger}_{\boldsymbol{k}\sigma} &= -i \int d^4x \Big\{ f_{\boldsymbol{k}\sigma}{}^{\mu}(x) \Box^x T(\mathcal{O} A_\mu(x)) \\
&\qquad -(1-\alpha)\, [\overline{h}_{\boldsymbol{k}\sigma}(x) \Box^x + f_{\boldsymbol{k}\sigma}{}^{\mu}(x) \partial^x_\mu]\, T(\mathcal{O} B(x)) \Big\} \;, \\
\mathcal{A}^{out}_{\boldsymbol{k}\sigma} T(\mathcal{O}) - T(\mathcal{O})\, \mathcal{A}^{in}_{\boldsymbol{k}\sigma} &= i \int d^4x \Big\{ f_{\boldsymbol{k}\sigma}{}^{\mu *}(x) \Box^x T(\mathcal{O} A_\mu(x)) \\
&\qquad -(1-\alpha)\, [\overline{h}^{\,*}_{\boldsymbol{k}\sigma}(x) \Box^x + f_{\boldsymbol{k}\sigma}{}^{\mu *}(x) \partial^x_\mu]\, T(\mathcal{O} B(x)) \Big\} \;,
\end{aligned}
\tag{A.28}
$$

and

$$
\begin{aligned}
\mathcal{B}^{out\dagger}_{\boldsymbol{k}} T(\mathcal{O}) - T(\mathcal{O})\, \mathcal{B}^{in\dagger}_{\boldsymbol{k}} &= -i \int d^4x g_{\boldsymbol{k}}(x) \Box^x T(\mathcal{O} B(x)) \;, \\
\mathcal{B}^{out}_{\boldsymbol{k}} T(\mathcal{O}) - T(\mathcal{O})\, \mathcal{B}^{in}_{\boldsymbol{k}} &= i \int d^4x g_{\boldsymbol{k}}{}^{*}(x) \Box^x T(\mathcal{O} B(x)) \;.
\end{aligned}
\tag{A.29}
$$

Taking the vacuum expectation values of (A.28), we obtain the LSZ formulas:

$$
\begin{aligned}
\langle 0 |\, T(\mathcal{O})\, | \boldsymbol{k}\sigma; in \rangle &= i \int d^4x \Big\{ f_{\boldsymbol{k}\sigma}{}^{\mu}(x) \Box^x \,\langle 0 |\, T(\mathcal{O} A_\mu(x))\, | 0 \rangle \\
&\qquad -(1-\alpha)\, [\, \overline{h}_{\boldsymbol{k}\sigma}(x) \Box^x + f_{\boldsymbol{k}\sigma}{}^{\mu}(x)\, \partial^x_\mu]\, \langle 0 |\, T(\mathcal{O} B(x))\, | 0 \rangle \Big\} \;, \\
\langle \boldsymbol{k}\sigma; out |\, T(\mathcal{O})\, | 0 \rangle &= i \int d^4x \Big\{ f_{\boldsymbol{k}\sigma}{}^{\mu *}(x) \Box^x \,\langle 0 |\, T(\mathcal{O} A_\mu(x))\, | 0 \rangle \\
&\qquad -(1-\alpha)\, [\, \overline{h}_{\boldsymbol{k}\sigma}{}^{*}(x) \Box^x + f_{\boldsymbol{k}\sigma}{}^{\mu *}(x)\, \partial^x_\mu]\, \langle 0 |\, T(\mathcal{O} B(x))\, | 0 \rangle \Big\} \;,
\end{aligned}
\tag{A.30}
$$

and

$$
\begin{aligned}
\langle 0 |\, T(\mathcal{O})\, | \boldsymbol{k}\, in \rangle &= i \int d^4x g_{\boldsymbol{k}}(x) \Box^x \,\langle 0 |\, T(\mathcal{O} B(x))\, | 0 \rangle \\
\langle \boldsymbol{k}\, out |\, T(\mathcal{O})\, | 0 \rangle &= i \int d^4x g_{\boldsymbol{k}}{}^{*}(x) \Box^x \,\langle 0 |\, T(\mathcal{O} B(x))\, | 0 \rangle
\end{aligned}
\tag{A.31}
$$

For the case of the Feynman gauge ($\alpha = 1$), $\langle 0 |\, T(\mathcal{O} B(x))\, | 0 \rangle$ terms in (A.30) disappear, so that we have much simpler formulas.

B Invariant Regularization and Finiteness for the Energy-Momentum Tensor

In this appendix, we show, although might be well-known as a "common" sense, that naïvely introduced cut-off,

$$\frac{1}{p^2} \longrightarrow \lim_{\Lambda\to\infty} \frac{1}{p^2}\left(\frac{-\Lambda^2}{p^2-\Lambda^2}\right)^N , \tag{B.1}$$

where N is some suitable number to make the whole integral finite, breaks the Lorentz invariance but the dimensional regularization does not; since there seems very few examples for demonstrating this "common" sense explicitly. In the subsequent section, finiteness for the energy-momentum tensor is argued.

B.1 A Need for Invariant Regularization

To simplify the discussion, consider the single scalar model described by

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\mu^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \\ &+\frac{(Z-1)}{2}\left(\partial_\mu\phi\partial^\mu\phi - \mu^2\phi^2\right) - \frac{\mu^2}{2}Z(Z_\mu-1)\phi^2 - \frac{\lambda}{4!}(Z_\lambda Z^2-1)\phi^4 ,\end{aligned} \tag{B.2}$$

where all quantities are renormalized and

$$\phi_{bare} = Z^{\frac{1}{2}}\phi,\ \mu^2_{bare} = Z_\mu\mu^2,\ \lambda_{bare} = Z_\lambda\lambda\,. \tag{B.3}$$

The energy-momentum tensor is,

$$\Theta^{\mu\nu} = Z\partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left(\frac{Z}{2}\partial^\lambda\phi\partial_\lambda - \frac{\mu^2}{2}ZZ_\mu\phi^2 - \frac{\lambda}{4!}Z^2Z_\lambda\phi^4\right) . \tag{B.4}$$

(For brevity's sake we do not consider an improvement; which accuires importance in the case of the trace identity [11].) By following the standard procedure [9], the Ward-Takahashi relation (WT) for the amputated Green's function, Γ, is found to be

$$\partial^x_\mu\Gamma^{\mu\nu}(x;y,z) + i\partial^\nu_x\delta^4(x-y)\,\Gamma(x,z) + i\partial^\nu_x\delta^4(x-z)\,\Gamma(x,y) = 0\,, \tag{B.5}$$

to give

$$(k+p)_\mu\Gamma^{\mu\nu}(k,p) + ik^\nu\Gamma(-p) + ip^\nu\Gamma(k) = 0\,, \tag{B.6}$$

where

$$\begin{aligned}\Gamma^{\mu\nu}(x;y,z) &= \int\frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\mathrm{e}^{-iky}\mathrm{e}^{-ipz}\mathrm{e}^{i(k+p)z}\Gamma^{\mu\nu}(k,p)\ ,\\ \Gamma(x,y) &= \int\frac{d^4k}{(2\pi)^4}\mathrm{e}^{-ik(x-y)}\Gamma(k)\,.\end{aligned} \tag{B.7}$$

With the use of the Feynman cut-off (B.1), $\Gamma(k)$ is obtained as

$$\Gamma(k) = -i(k^2-\mu^2) - \left\{\Sigma(k^2) + i(Z-1)(k^2-\mu^2) - i\mu^2Z(Z_\mu-1)\right\}\ , \tag{B.8}$$

with

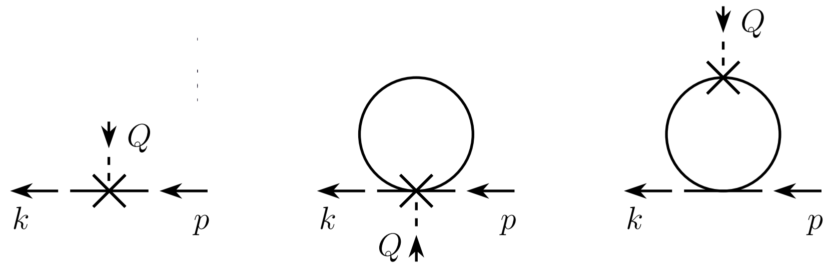
$$\varSigma(k^2) = \varSigma(0) = \frac{-i\lambda}{2}\int \frac{d^4l}{(2\pi)^4}\frac{i}{l^2-\mu^2}\left(\frac{-\Lambda^2}{l^2-\Lambda^2}\right)^2 = \frac{-i\lambda}{2(4\pi)^2}\mu^2\left[\frac{\Lambda^2}{\mu^2} - \ln\left(\frac{\Lambda^2}{\mu^2}\right) + 1\right]. \tag{B.9}$$

Put $Z = 1$ and choose Z_μ such that $\varSigma(0) - i\mu^2(Z_\mu - 1)$ be finite. Therefore

$$\varGamma(k) = -i(k^2 - \mu^2) - \left\{\varSigma(0) - i\mu^2(Z_\mu - 1)\right\}. \tag{B.10}$$

As for $\varGamma^{\mu\nu}(k,p)$, since $Z_\lambda = 1$ and $Z = 1$,

$\varGamma^{\mu\nu}(k,p) =$



(B.11)

$$= -[k^\mu p^\nu + k^\nu p^\mu - g^{\mu\nu}kp] + g^{\mu\nu}\mu^2 Z_\mu + g^{\mu\nu}\frac{\lambda}{2}\int\frac{d^4l}{(2\pi)^4}\frac{i}{l^2-\mu^2}\left(\frac{-\Lambda^2}{l^2-\Lambda^2}\right)^2$$

$$+\frac{i\lambda}{2}\int\frac{d^4l}{(2\pi)^4}\frac{l^\mu(l-q)^\nu + l^\nu(l-q)^\mu - g^{\mu\nu}l(l-q) + g^{\mu\nu}\mu^2}{(l^2-\mu^2)[(l-q)^2-\mu^2]}\left(\frac{-\Lambda^2}{l^2-\Lambda^2}\right)^2,$$

where $q = k + p$. The integral in the first line is just the same as $\varSigma(0)$, then WT (B.6) reads

$$\begin{aligned}
&(k+p)_\mu\varGamma^{\mu\nu}(k,p) + ik^\nu\varGamma(-p) + ip^\nu\varGamma(k) \\
&= \frac{i\lambda}{2}q_\mu\int\frac{d^4l}{(2\pi)^4}\frac{l^\mu(l-q)^\nu + l^\nu(l-q)^\mu - g^{\mu\nu}l(l-q) + g^{\mu\nu}\mu^2}{(l^2-\mu^2)[(l-q)^2-\mu^2]}\left(\frac{-\Lambda^2}{l^2-\Lambda^2}\right)^2 \\
&= -\frac{q^\nu}{4(4\pi)^2}\Lambda^2 + O(\Lambda^0) \neq 0\,.
\end{aligned} \tag{B.12}$$

Therefore WT (B.6) cannot be met by the Feynman cut-off scheme (B.1), which implies that the translational invariance is broken in this regularization. On the contrary adopting the dimensional regularization, we have, instead of (B.13),

$$\begin{aligned}
&(k+p)_\mu\varGamma^{\mu\nu}(k,p) + ik^\nu\varGamma(-p) + ip^\nu\varGamma(k) \\
&= \frac{i\lambda}{2}q_\mu\int\frac{d^nl}{(2\pi)^n}\frac{l^\mu(l-q)^\nu + l^\nu(l-q)^\mu - g^{\mu\nu}l(l-q) + g^{\mu\nu}\mu^2}{(l^2-\mu^2)[(l-q)^2-\mu^2]} \\
&= \frac{i\lambda}{2}\int\frac{d^nl}{(2\pi)^n}\left\{\frac{(l-q)^\nu}{(l-q)^2-\mu^2} - \frac{l^\nu}{l^2-\mu^2}\right\} = 0\,,
\end{aligned} \tag{B.13}$$

because of the fact that we are free to make a shift of the loop momentum in the dimensional regularization.

B.2 Cancellation of the Divergence in $G^{\mu\nu;\lambda\kappa}(q,q')$

Finiteness of the energy-momentum tensor has already been proven in [9], and QED is indeed the case. To see this, we here show the cancellation of divergences in

$$G^{\mu\nu;\lambda\kappa}(q,q') \equiv G_{\mathrm{g}}^{\mu\nu;\lambda\kappa}(q,q') + G_{\mathrm{m}}^{\mu\nu;\lambda\kappa}(q,q')\,. \tag{B.14}$$

Up to the one-loop, in view of Fig.3, it gives

$$
\begin{aligned}
&G^{\mu\nu;\lambda\kappa}(q,q') = \frac{-i}{q^2}X^{\mu\nu\lambda\kappa}(q,q')\frac{-i}{q'^2} \\
&+\frac{-i}{q^2}X^{\mu\nu\lambda\rho}(q,q')\frac{-i}{q'^2}\Pi_{\rho\sigma}(q')\frac{-i}{q'^2}d^{\sigma\kappa}(q') + \frac{-i}{q^2}d^{\lambda\rho}(q)\Pi_{\rho\sigma}(q)\frac{-i}{q^2}X^{\mu\nu\sigma\kappa}(q,q')\frac{-i}{q'^2} \\
&+\frac{-i}{q^2}d^{\lambda}{}_{\rho}(q)\Pi^{\mu\nu\rho\sigma}(q,q')d_{\sigma}{}^{\kappa}(q')\frac{-i}{q'^2} \ ,
\end{aligned}
\tag{B.15}
$$

whose expressions from the first to the final line come from (2.30), (2.42), and (2.45) respectively.

The Lagrangian is now the Nakanishi-Lautrup one in the fully renormalized form,

$$
\begin{aligned}
\mathcal{L} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A^{\mu}\,\partial_{\mu}B + \frac{\alpha}{2}B^2 + \overline{\psi}(\frac{i}{2}\overleftrightarrow{\partial\!\!\!/} - m)\psi + e\overline{\psi}A\!\!\!/\psi \\
&-(Z_3-1)\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (Z_2-1)\overline{\psi}(\frac{i}{2}\overleftrightarrow{\partial\!\!\!/} - m)\psi \\
&-(Z_m-1)Z_2 m\overline{\psi}\psi + (Z_1-1)e\overline{\psi}A\!\!\!/\psi \ ,
\end{aligned}
\tag{B.16}
$$

with the relation between the bare and the renormalized quantities:

$$
\begin{aligned}
&A^{\mu}_{bare} = Z_3^{\frac{1}{2}}A^{\mu} \ , \quad B_{bare} = Z_3^{-\frac{1}{2}}B \ , \quad \psi_{bare} = Z_2^{\frac{1}{2}}\psi \ , \\
&\alpha_{bare} = Z_3\alpha \ , \quad e_{bare} = Z_1 Z_2^{-1}Z_3^{-1/2}e \ , \quad m_{bare} = Z_m m \ .
\end{aligned}
\tag{B.17}
$$

The energy-momentum tensor is therefore found as

$$
\begin{aligned}
\Theta^{\mu\nu} = &-Z_3\left\{F^{\mu\rho}F^{\nu}{}_{\rho} - \frac{g^{\mu\nu}}{4}F^{\rho\sigma}F_{\rho\sigma}\right\} \\
&-(A^{\mu}\,\partial^{\nu}B + A^{\nu}\,\partial^{\mu}B) - g^{\mu\nu}\left\{\frac{\alpha}{2}B^2 - A^{\rho}\partial_{\rho}B\right\} \\
&+Z_2\left\{\frac{i}{4}\overline{\psi}(\gamma^{\mu}\overleftrightarrow{\partial^{\nu}} + \gamma^{\nu}\overleftrightarrow{\partial^{\mu}})\psi - g^{\mu\nu}\overline{\psi}\left(\frac{i}{2}\overleftrightarrow{\partial\!\!\!/} - Z_m m\right)\psi\right\} \\
&+eZ_1\left\{\frac{1}{2}\overline{\psi}(\gamma^{\mu}A^{\nu} + \gamma^{\nu}A^{\mu})\psi - g^{\mu\nu}\overline{\psi}A\!\!\!/\psi\right\} \ .
\end{aligned}
\tag{B.18}
$$

In view of (B.15) divergences lies in the vacuum polarization, $\Pi^{\mu\nu}(q)$, and $\Pi^{\mu\nu;\rho\sigma}(q,q')$. As usual we remove the divergence of $\Pi^{\mu\nu}(q)$: the photon propagator up to the one loop reads

$$
\begin{aligned}
&\int d^4x e^{iqx}\,\langle 0|\,TA^{\mu}(x)A^{\nu}(0)\,|0\rangle \\
&= \frac{-i}{q^2}d^{\lambda\kappa}(q) + \frac{-i}{q^2}(g^{\lambda\kappa}q^2 - q^{\lambda}q^{\kappa})\{\Pi(q) - i(Z_3-1)\}\frac{-i}{q^2} \ ,
\end{aligned}
\tag{B.19}
$$

where $\Pi(q)$ has been given in (2.43),

$$
\Pi(q) = -ie^2\frac{2\,\mathrm{tr}\mathbf{1}}{(4\pi)^2}\Gamma(2-\frac{n}{2})\int_0^1 dx\ x(1-x)\left(\frac{m^2 - x(1-x)q^2}{4\pi}\right)^{\frac{n}{2}-2} \ .
\tag{B.20}
$$

Z_3 is chosen to cancel out the divergent part of $\Pi(q)$. While the superficial degree of divergence for $\Pi^{\mu\nu;\lambda\kappa}(q,q')$ is two so that the Taylor expansion in the expression in (2.46) gives

$$
\Pi^{\mu\nu;\lambda\kappa}(q,q') = i\Pi(0)\widetilde{X}^{\mu\nu\lambda\kappa}(q,q') + O((q,q')^3) \ ,
\tag{B.21}
$$

where $O((q,q')^3)$ is finite.

Now recall that (B.15) is rewritten in terms of the renormalized form such as

$$
\begin{aligned}
G^{\mu\nu;\lambda\kappa}(q,q') = & \frac{-i}{q^2} Z_3 \widetilde{X}^{\mu\nu\lambda\kappa}(q,q') \frac{-i}{q'^2} \\
& + \frac{-i}{q^2} \{ \alpha\, q^\lambda q'^\kappa g^{\mu\nu} - q^\lambda X^{\mu\nu\kappa}(q,q') - q'^\kappa X^{\mu\nu\lambda}(q',q) \} \frac{-i}{q'^2} \\
& + \frac{-i}{q^2} X^{\mu\nu\lambda\rho}(q,q') \frac{-i}{q'^2} (q'^2 g_{\rho\sigma} - q'_\rho q'_\sigma) \{ \Pi(q') - i(Z_3 - 1) \} d^{\sigma\kappa}(q') \frac{-i}{q'^2} \\
& + \frac{-i}{q^2} d^{\lambda\rho}(q) (q^2 g_{\rho\sigma} - q_\rho q_\sigma) \{ \Pi(q) - i(Z_3 - 1) \} \frac{-i}{q^2} X^{\mu\nu\sigma\kappa}(q,q') \frac{-i}{q'^2} \\
& + \frac{-i}{q^2} \Pi^{\mu\nu;\lambda\kappa}(q,q') \frac{-i}{q'^2} \ ,
\end{aligned}
\tag{B.22}
$$

where use has been made of the transversal condition of $\Pi^{\mu\nu;\lambda\kappa}(q,q')$ (2.47) in the final line. Note that it has a finite combination $\Pi(q) - iZ_3$; since

$$
\text{the first term + the last term} = \frac{-i}{q^2} i(\Pi(0) - iZ_3) \widetilde{X}^{\mu\nu\lambda\kappa}(q,q') \frac{-i}{q'^2} + O((q,q')^3) \ . \tag{B.23}
$$

Therefore there are no divergences in $G^{\mu\nu;\lambda\kappa}(q,q')$.

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